Chapter 8 Antenna Theory

8-1 Calculation of EM Fields of Antennas

L. J. Chu (朱蘭成)： 朱蘭成院士 1913 年 8 月 24 日生於江蘇淮陰，1934 年畢業於上海交通大學，1935 與 1938 年獲美國麻省理工學院碩士和博士學位，畢業後在麻省理工學院任教。1958 年當選第二屆中研院院士，是電磁波及雷達研究方面的三大國際權威之一，台大電機 1 館是由朱蘭成號召募資興建。1973 年逝世。

Stratton-Chu formulas for calculating EM Fields of antennas: (by L. J. Chu)

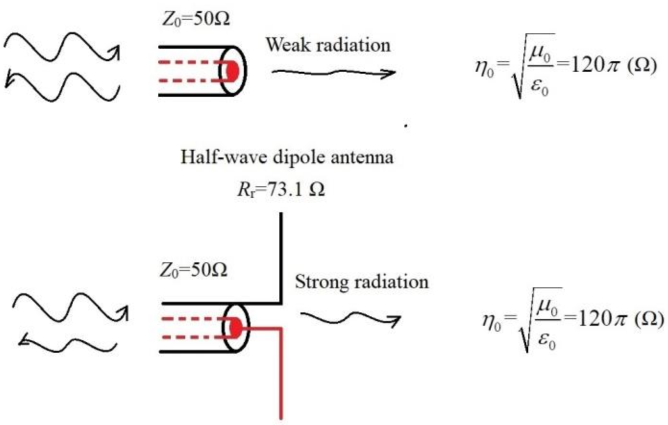
$$\vec{E}(\vec{r}) = \iiint_{V'}[-j\omega\mu G\vec{J} + \frac{\rho}{\varepsilon}\nabla' G - \vec{J}_m \times \nabla' G]dV' + \oiint_{S'}[-j\omega\mu G(\hat{a}_n \times \vec{H}) + (\hat{a}_n \cdot \vec{E})\nabla' G + (\hat{a}_n \times \vec{E}) \times \nabla' G]dS'$$

$$\vec{H}(\vec{r}) = \iiint_{V'}[-j\omega\varepsilon G\vec{J}_m + \frac{\rho_m}{\mu}\nabla' G + \vec{J} \times \nabla' G]dV' + \oiint_{S'}[j\omega\varepsilon G(\hat{a}_n \times \vec{E}) + (\hat{a}_n \cdot \vec{H})\nabla' G + (\hat{a}_n \times \vec{H}) \times \nabla' G]dS'$$

where $G = \dfrac{e^{-jkr}}{4\pi r}$ is Green's function in the free space.

Y. T. Lo (羅遠梓): Yuen Tze Lo (MSEE'49, PhD'52) died in 2002. He was 82. Lo invented the broadband television receiving antenna, and he developed the cavity model theory for microstrip patch antennas now used in global positioning systems (GPS). In 1986, Lo was elected to the National Academy of Engineering for inventions and innovative ideas that significantly advanced the theory and design of antennas and arrays.

J. A. Kong (孔金甌)： 孔教授 1942 年出生於中國江蘇，2008 年去世。孔教授是孔子第 74 代孫，為台大電機學士，交大碩士，美國 Syracuse 大學博士，他自 1969 年任教麻省理工學院電機系，為電磁波泰斗，曾出版 30 本電磁學著作和 700 篇研究論文。他曾參與阿波羅十七登月計劃及探測月球表面與內部物質的設計研究；解決了雙子星電視干擾問題，發展和拓寬了微波遙測理論模式。



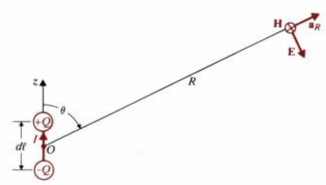
Case 1 Given $\vec{J}(x,y,z,t)$, $\vec{A}=\frac{\mu}{4\pi}\iiint_{v'}\frac{\vec{J}e^{-jkR}}{R}dv'$, $\vec{H}=\frac{1}{\mu}\nabla\times\vec{A}$, $\vec{E}=\frac{1}{j\omega\varepsilon}\nabla\times\vec{H}$

Case 2 Given $\rho(x,y,z,t)$, $V=\frac{1}{4\pi\varepsilon}\iiint_{V'}\frac{\rho e^{-jkR}}{R}dv'$, $\nabla\cdot\vec{J}=-\frac{\partial\rho}{\partial t}$, $\vec{E}=-\nabla V-\frac{\partial\vec{A}}{\partial t}=\frac{1}{j\omega\varepsilon}\nabla\times\vec{H}$

Eg. Assume the current on a very thin center-fed half-wave dipole lying along the *z*-axis to be $I_0\cos(\beta z)$, where $\beta=2\pi/\lambda$. Find the charge distribution on the dipole.

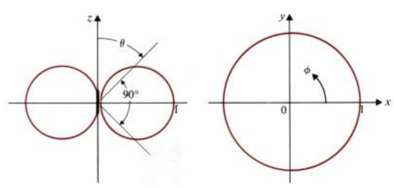
(Sol.) $\nabla\cdot\vec{J}=-j\omega\rho\Rightarrow\rho=\frac{j}{\omega}\frac{dI(z)}{dz}=-j\frac{\beta}{\omega}I_0\sin\beta z$

Elemental electrical dipole (Hertzian dipole):



$$\vec{p}=\hat{z}Qd\ell$$

$$I=\pm\frac{dQ}{dt}=\pm j\omega Q,\quad Q=\pm\frac{I}{j\omega}$$



(a) *E*-plane pattern. (b) *H*-plane pattern.

$$\Rightarrow\vec{A}=\hat{z}\frac{\mu_0 Id\ell}{4\pi}\cdot\frac{e^{-j\beta R}}{R}=(\hat{a}_R\cos\theta-\hat{a}_\theta\sin\theta)\frac{\mu_0 Id\ell}{4\pi}\cdot\frac{e^{-j\beta R}}{R}$$

$$=\hat{a}_R A_R+\hat{a}_\theta A_\theta+\hat{a}_\phi A_\phi$$

$$\Rightarrow\begin{cases}A_R=A_z\cos\theta=\frac{\mu_0 Id\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\cos\theta\\ A_\theta=-A_z\sin\theta=-\frac{\mu_0 Id\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\sin\theta\\ A_\phi=0\end{cases}\Rightarrow$$

$$\vec{H}=\frac{1}{\mu_0}\nabla\times A=\hat{a}_\phi\frac{1}{\mu_0 R}\left[\frac{\partial}{\partial R}(RA_\phi)-\frac{\partial A_R}{\partial\theta}\right]$$
$$=-\hat{a}_\phi\frac{Id\ell}{4\pi}\beta^2\sin\theta\cdot\left[\frac{1}{j\beta R}+\frac{1}{(j\beta R)^2}\right]e^{-j\beta R}$$

$$\vec{E}=\frac{1}{j\omega\varepsilon_0}\nabla\times\vec{H}=\frac{1}{j\omega\varepsilon_0}\left[\hat{a}_R\frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(H_\phi\sin\theta)-\hat{a}_\theta\frac{1}{R}\frac{\partial}{\partial R}(RH_\phi)\right]$$

$$\Rightarrow\begin{cases}E_R=-\frac{Id\ell}{4\pi}\eta_0\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2}+\frac{1}{(j\beta R)^3}\right]e^{-j\beta R}\\ E_\theta=-\frac{Id\ell}{4\pi}\eta_0\beta^2\sin\theta\left[\frac{1}{j\beta R}+\frac{1}{(j\beta R)^2}+\frac{1}{(j\beta R)^3}\right]e^{-j\beta R}\\ E_\phi=0,\ where\ \eta_0=\sqrt{\mu_0/\varepsilon_0}\cong120\pi(\Omega)\end{cases}$$

Far field of a Hertzian dipole: if $\beta R=2\pi R/\lambda>>1$

$$H_\phi=j\frac{Id\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta,\quad E_\theta=j\frac{Id\ell}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta$$

Elemental magnetic dipoles: $\vec{m} = \hat{z}IS = \hat{z}m$

$$\vec{m} = \hat{z}I\pi b^2 = \hat{z}m \Rightarrow \vec{A} = \frac{\mu_0 I}{4\pi}\oint \frac{e^{-j\beta R_1}}{R_1} d\vec{\ell}'$$

$$e^{-j\beta R_1} = e^{-j\beta R} e^{-j\beta(R_1 - R)} \cong e^{-j\beta R}[1 - j\beta(R_1 - R)]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} e^{-j\beta R}[(1 + j\beta R)\oint \frac{d\vec{\ell}'}{R_1} - j\beta \oint d\vec{\ell}'] \Rightarrow \vec{A} = \hat{a}_\phi \frac{\mu_0 m}{4\pi R^2}(1 + j\beta R)e^{-j\beta R} \sin\theta$$

$$\vec{H} = \frac{1}{\mu_0}\nabla\times\vec{A} \quad \text{and} \quad \vec{E} = \frac{1}{j\omega\varepsilon_0}\nabla\times\vec{H}$$

$$\Rightarrow \begin{cases} E_\phi = \dfrac{j\omega\mu_0 m}{4\pi}\beta^2 \sin\theta[\dfrac{1}{j\beta R} + \dfrac{1}{(j\beta R)^2}]e^{-j\beta R} \\ H_R = -\dfrac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2 2\cos\theta[\dfrac{1}{(j\beta R)^2} + \dfrac{1}{(j\beta R)^3}]e^{-j\beta R} \\ H_\theta = -\dfrac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2 \sin\theta[\dfrac{1}{j\beta R} + \dfrac{1}{(j\beta R)^2} + \dfrac{1}{(j\beta R)^3}]e^{-j\beta R} \end{cases}$$

Far field of an elemental magnetic dipoles:

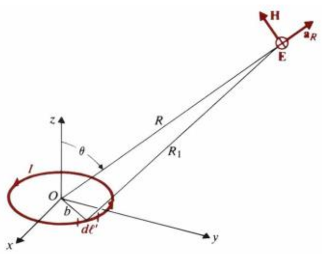
$$E_\phi = \frac{\omega\mu_0 m}{4\pi}(\frac{e^{-j\beta R}}{R})\beta\sin\theta, \quad H_\theta = -\frac{\omega\mu_0 m}{4\pi\eta_0}(\frac{e^{-j\beta R}}{R})\beta\sin\theta$$

Eg. A small filamentary rectangular loop of dimensions L_x and L_y lies in the *xy*-plane with its center at the origin and sides parallel to the *x*- and *y*-axes. The loop carries a current $i(t)=I_0\cos(\omega t)$. Assuming L_x and L_y to be much less than the wavelength, find the expressions for the following quantities at a point in the far zone: (a) vector magnetic potential, (b) electric field intensity, (c) magnetic field intensity.

(Sol.) (a) $\vec{m} = \hat{z}I_0\cos\omega t L_x L_y$, $\vec{A} = \hat{a}_\phi \dfrac{\mu_0 I_0 \cos\omega t L_x L_y}{4\pi R}(1 + j\beta R)e^{-j\beta R}\cdot\sin\theta$

(b) $E_\phi = \dfrac{\omega\mu_0 I_0\cos\omega t L_x L_y}{4\pi}(\dfrac{e^{-j\beta R}}{R})\beta\sin\theta$

(c) $H_\theta = -\dfrac{\omega\mu_0 I_0\cos\omega t L_x L_y}{4\pi\eta_0}(\dfrac{e^{-j\beta R}}{R})\beta\sin\theta$



Eg. A composite antenna consists of an elemental Hertzian electric dipole of length L along the z-axis and an elemental magnetic dipole of area S lying in the xy-plane. Equal time-harmonic currents of amplitude I_0 and angular frequency ω flow in the dipoles. (a) Verify that the far field of the composite antenna is elliptically polarized. (b) Determine the condition for circular polarization.

(Sol.) (a) $\vec{E} = \hat{a}_\theta \dfrac{jI_0L}{4\pi}\eta_0\beta\sin\theta\dfrac{e^{-j\beta R}}{R} + \hat{a}_\phi\dfrac{\omega\mu_0 I_0 S}{4\pi\eta_0}\beta\sin\theta\dfrac{e^{-j\beta R}}{R} = \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$

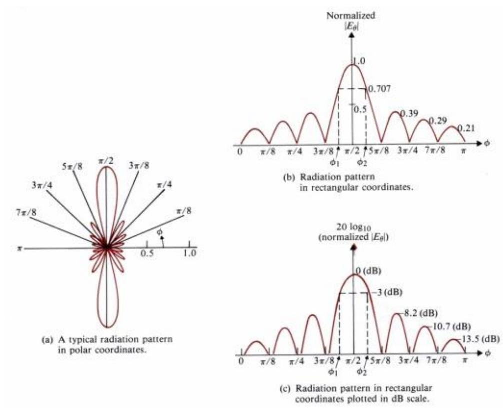
(b) $E_\theta = E_\phi \Rightarrow \eta_0 L = \dfrac{\omega\mu_0 S}{\eta_0}$

Duality between elemental electric and magnetic dipoles: (E_e, H_e) due to electric dipole and (E_m, H_m) due to magnetic dipole

$$E_e \leftrightarrow \eta_0 H_m,\ \ H_e \leftrightarrow -\frac{E_m}{\eta_0},\ \ Id\ell \leftrightarrow j\beta m = j\beta IS,\ \ d\ell \leftrightarrow \beta S$$

8-2 Radiation Patterns of Antennas

Half-power beam width: Angular width of main beam between the half-power (-3dB) points



Sidelobe level: (|E_{max}| in one sidelobe)/(|E_{max}| in main beam)

Null positions: Directions which have no radiations in the far-field zone.

Directivity: $D=\dfrac{4\pi U_{max}}{P_r} = \dfrac{4\pi\left|\vec{E}_{max}\right|^2}{\int_0^{2\pi}\int_0^{\pi}\left|\vec{E}(\theta,\phi)\right|^2\sin\theta d\theta d\phi}$, where

$U=R^2P_{av} \propto R^2\left|\vec{E}\right|^2$

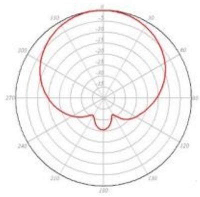
and $P_r= \oint P_{av}dS = \oint Ud\Omega \propto R^2\int_0^{2\pi}\int_0^{\pi}\left|\vec{E}\right|^2\sin\theta d\theta d\phi$ is the time-average radiated power

Directivity gain: $G_D(\theta,\phi)= \dfrac{4\pi U(\theta,\phi)}{P_r} = \dfrac{4\pi\left|\vec{E}(\theta,\phi)\right|^2}{\int_0^{2\pi}\int_0^{\pi}\left|\vec{E}(\theta,\phi)\right|^2\sin\theta d\theta d\phi}$, ∴ $D=(G_D)_{max}$

Power gain: $G_P = \dfrac{4\pi U_{max}}{P_i}$, where $P_i= P_r+P_l$, P_i: total input power, P_l: loss

Radiation efficiency: $\eta_r= G_P/D=P_r/P_i$

Note: **The half-power beam width of the antenna for broadcasting or wireless communication is wide but its directivity is low. Contrarily, the half-power beam width of the Radar antenna for detecting targets is narrow but its directivity is high.**



Antenna for broadcasting or wireless communication

Radar antenna for detecting targets

Eg. Find the directive gain and the directivity of a Hertzian dipole.

(Sol.) $P_{av} = \frac{1}{2}\mathrm{Re}|E \times H^*| = \frac{1}{2}|E_\theta||H_\phi|$, $U = \frac{(I d\ell)^2}{32\pi^2}\eta_0 \beta^2 \sin^2\theta$.

$$G_D(\theta,\phi) = \frac{4\pi \sin^2\theta}{\int_0^{2\pi}\int_0^{\pi}(\sin^2\theta)\sin\theta d\theta d\phi} = \frac{3}{2}\sin^2\theta, \quad D = G_D(\frac{\pi}{2},\phi) = 1.5 = 1.76\ (dB).$$

Eg. Find the radiation resistance of a Hertzian dipole.

(Sol.) $P_r = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi} E_\theta H_\phi^* R^2 \sin\theta d\theta d\phi$

$$= \frac{I^2 (d\ell)^2}{32\pi^2}\eta_0\beta^2\int_0^{2\pi}\int_0^{\pi}\sin^3\theta d\theta d\phi = \frac{I^2(d\ell)^2}{12\pi}\eta_0\beta^2 = \frac{I^2}{2}[80\pi^2(\frac{d\ell}{\lambda})^2] = \frac{I^2}{2}R_r$$

$$\therefore\ R_r = 80\pi^2(\frac{d\ell}{\lambda})^2$$

Eg. Find the radiation efficiency of an isolated Hertzian dipole made of a metal wire of radius *a*, length *d*, and conductivity σ.

(Sol.) The ohmic power loss is $P_\ell = \frac{1}{2}I^2 R_\ell$. The radiated power is $P_r = \frac{1}{2}I^2 R_r$

$$\eta_r = \frac{P_r}{P_r + P_\ell} = \frac{1}{1 + (R_\ell / R_r)}, \quad R_\ell = R_s(\frac{d\ell}{2\pi a}),$$

where $R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} \Rightarrow \eta_r = \frac{1}{1 + \frac{R_s}{160\pi^3}(\frac{\lambda}{a})(\frac{\lambda}{d\ell})}$

Assume that a=1.8mm, $d\ell = 2m$, $f = 1.5MHz$, and $\sigma = 5.80 \times 10^7 (S/m)$

$\lambda = \frac{c}{f} = 200\,(m)$, $R_s = 3.20 \times 10^{-4}\,(\Omega)$, $R_\ell = 0.057\,(\Omega)$, $R_r = 0.079(\Omega)$,

and $\eta_r = 58\%$

Eg. A 1*MHz* uniform current flows in a vertical antenna of the length 15*m*. The antenna is a center-fed copper rod having a radius of 2*cm*. Find (a) the radiation resistance, (b) the radiation efficiency, (c) the maximum electric field intensity at a distance of 20*km*, the radiated power of the antenna is 1.6*kW*.

(Sol.) $\lambda = \dfrac{3\times10^8}{10^6} = 300\,m >> 15m = d\ell$, a=0.02m, σ_{copper}=5.8×10^7,

$$R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = 2.6\times10^{-4}$$

(a) $R_r = 80\pi^2(15/300)^2 = 1.97\Omega$, (b) $\eta_r = 1/(1+\dfrac{R_s}{160\pi^3(\lambda/a)(\lambda/d\ell)}) = 98\%$

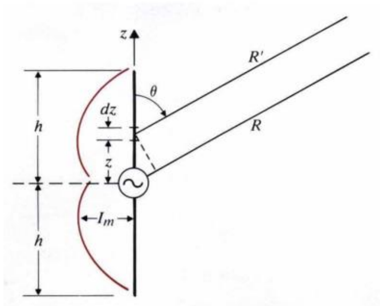
(c) $P_r = \dfrac{I^2(d\ell)^2}{12\pi}\eta_0\beta^2 = 1600 \Rightarrow |E_\theta|_{\max} = (\dfrac{Id\ell}{4\pi})\dfrac{\eta_0\beta}{R} \approx 1.9\times10^{-2}V/m$

Eg. A time-harmonic uniform current I_0cos(ω*t*) flows in a small circular loop of radius *b*(<<λ) lying in the *xy*-plane. (a) Find the radiation resistance R_r of the magnetic dipole. (b) Obtain an expression for its radiation efficiency η_r if the loop is made of radius *a*.

(Sol.) (a) Duality $\Rightarrow d\ell \leftrightarrow \beta\pi b^2 \Rightarrow R_r = 80\pi^2(\dfrac{\beta\pi b^2}{\lambda})^2 = 320\pi^6(\dfrac{b}{\lambda})^4$

(b) $$\eta_r = \frac{1}{1+\dfrac{R_s}{160\pi^3}(\dfrac{\lambda}{a})(\dfrac{\lambda}{\beta\pi b^2})}$$

8-3 Linear Dipole Antennas and Effective Lengths



Assume $I(z)=I_m \sin\beta(h-|z|)=\begin{cases} I_m \sin\beta(h-z), z>0 \\ I_m \sin\beta(h+z), z<0 \end{cases}$

$$E_\theta=\eta_0 H_\varphi=\eta_0\int_{-h}^{h}\frac{I(z)dz}{4\pi}\frac{e^{-j\beta R'}}{R'}\beta\sin\theta$$

$(R>>h,\ \ R'=(R^2+z^2-2Rz\cos\theta)^{1/2}\approx R-z\cos\theta)$

$$\Rightarrow E_\theta\approx j\frac{I_m\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\int_{-h}^{h}\sin\beta(h-|z|)e^{j\beta z\cos\theta}dz=\frac{j60I_m}{R}\cdot e^{-j\beta R}\cdot F(\theta)$$

where $F(\theta)=\dfrac{\cos(\beta h\cos\theta)-\cos\beta h}{\sin\theta}$

Half-wave dipole: $2h=\lambda/2$, $\beta h=\pi/2$

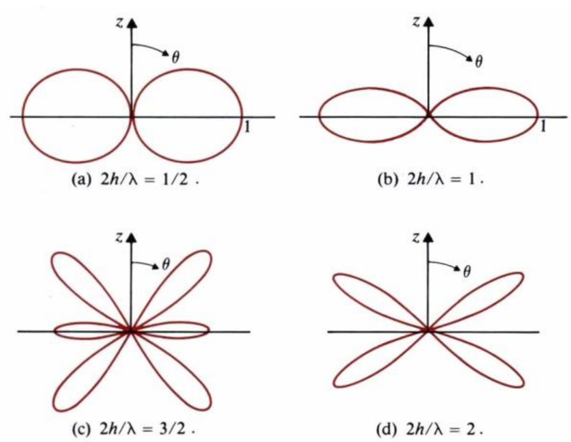
$$\begin{cases} E_\theta=\dfrac{j60I_m}{R}e^{-j\beta R}\left\{\dfrac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\} \\ H_\phi=\dfrac{jI_m}{2\pi R}e^{-j\beta R}\left\{\dfrac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\} \end{cases}\Rightarrow P_{av}=\frac{1}{2}E_\theta H_\phi^*=\frac{15I_m^2}{\pi R^2}\left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}^2$$

Half-power beam width of a half-wave dipole: $\Delta\theta=\theta_1-\theta_2\approx 78°$, where θ_1 and θ_2 are two roots of $\dfrac{\cos((\pi/2)\cos\theta)}{\sin\theta}=\dfrac{1}{\sqrt{2}}$.

$$P_r=\int_0^{2\pi}\int_0^{\pi}P_{av}R^2\sin\theta d\theta d\phi=30I_m^2\int_0^{\pi}\frac{\cos^2[(\pi/2)\cos\theta]}{\sin\theta}d\theta=36.54I_m^2(w)=\frac{I_m^2}{2}R_r$$

$\Rightarrow$ R_r=73.1Ω and $U_{max}=R^2P_{av}(\theta=\pi/2)=\dfrac{15}{\pi}I_m^2\Rightarrow D=\dfrac{4\pi U_{max}}{P_r}$=1.64>1.5

Radiation patterns of linear dipoles:



E-plane radiation patterns for center-fed dipole antennas

Effective length of a transmitting linear dipole antenna, $l_e(\theta)$:

$$E_\theta = \eta_0 H_\phi = j\frac{I_m \eta_0 \beta \sin\theta}{4\pi R} e^{-j\beta R}\int_{-h}^{h} \sin\beta(h-|z|)e^{j\beta z\cos\theta}dz = \frac{j30}{R}\beta e^{-j\beta R}\left\{\sin\theta\int_{-h}^{h} I(z)e^{j\beta z\cos\theta}dz\right\}$$

$= \frac{j30I(0)}{R}\beta e^{-j\beta R}\ell_e(\theta)$, where $l_e(\theta)=\frac{\sin\theta}{I(0)}\int_{-h}^{h} I(z)e^{j\beta z\cos\theta}dz$ is the **effective length**.

Maximum of $l_e(\theta)$ occurs when $\theta=\pi/2 \Rightarrow l_e(\theta=\pi/2)=\frac{1}{I(0)}\int_{-h}^{h} I(z)dz$

Note: $l_e=-V_{oc}/E_i$ is the effective length of a receiving linear dipole antenna = that of transmitting one.

Eg. Assume a sinusoidal current distribution on a center-fed, thin, straight half-wave dipole. Find its effective length. What is its maximum value?

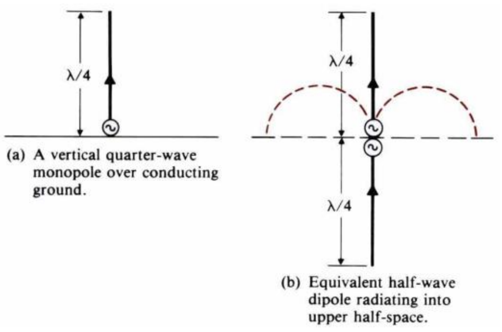
(Sol.) $I(0)=I_m$, $h=\lambda/4$,

$$\ell_e(\theta) = \frac{\sin\theta}{I(0)}\int_{-\lambda/4}^{\lambda/4} I_m \sin\beta(\frac{\lambda}{4}-|z|)e^{j\beta z\cos\theta}dz = \frac{2}{\beta}[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}],\quad \ell_e(\frac{\pi}{2}) = \frac{2}{\beta} = \frac{\lambda}{\pi}$$

Eg. A 1.5*MHz* uniform plane wave having a peak electric field intensity E_0 is incident on a half-wave dipole at an angle θ. (a) Find the expression for the open-circuit voltage V_{oc} at the terminals of the dipole. (b) If the dipole is connected to a matched load, what is the maximum power P_L delivered to the load?

(Sol.) (a) $V_{oc} = -E_0\ell_e = -\frac{\lambda E_0}{\pi}[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}]$, $\lambda = 200m$

(b) $P_L = \frac{1}{2}\left|\frac{V_{0c}}{R_r + R_L}\right|^2 R_L = \frac{V_{oc}^2}{8R_r}$



(a) A vertical quarter-wave monopole over conducting ground.
(b) Equivalent half-wave dipole radiating into upper half-space.

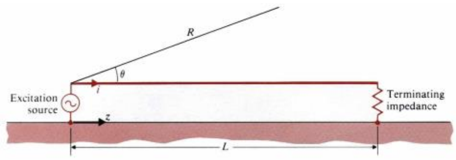
Monopole antenna: $P_r=18.27I_m^2W$, and $R_r=2P_r/I_m^2=36.54\Omega$ is exactly one-half of the radiation resistance of a half-wave antenna in the free space. $D=2\pi U_{max}/P_r=1.64$ is the same as the directivity of a half-wave antenna.

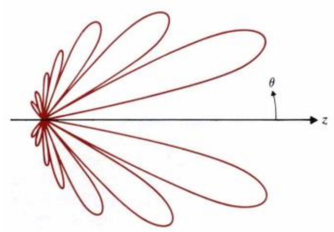
8-4 Traveling–wave Antenna

$I(z) = I_0 e^{-j\beta z}$, $dE_0 = j\frac{Idz}{4\pi}(\frac{e^{-j\beta R}}{R})\eta_0 \beta \sin\theta$ for a small dipole Idz

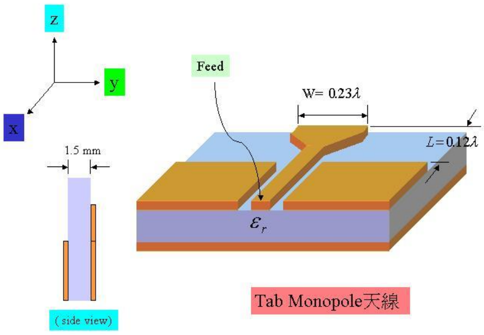
$E_\theta = \frac{j\eta_0 \beta \sin\theta}{4\pi r} e^{-j\beta R} \int_0^L I(z) e^{j\beta z \cos\theta} dz = \frac{j60 I_0}{R} e^{-j\beta[R+(L/2)(1-\cos\theta)]} F(\theta)$, where

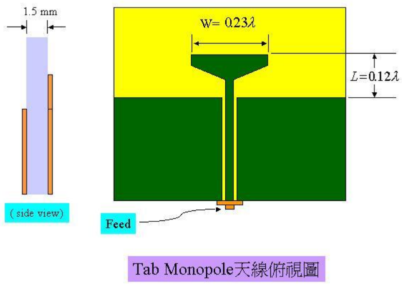
$$F(\theta) = \frac{\sin\theta \sin[\beta L(1-\cos\theta)/2]}{1-\cos\theta}$$

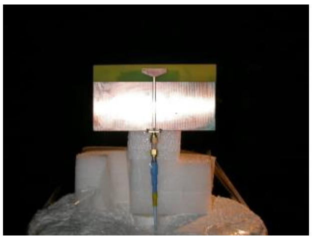
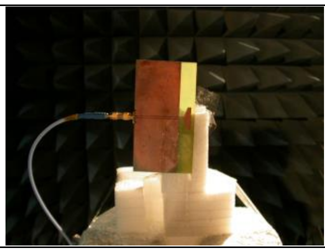
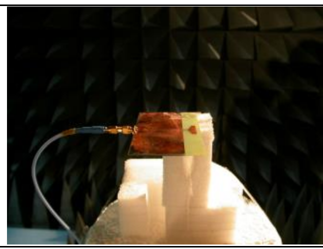




Some examples of coplanar antennas (by H. –C. Chen and Dr. I-Fong Chen)**:**





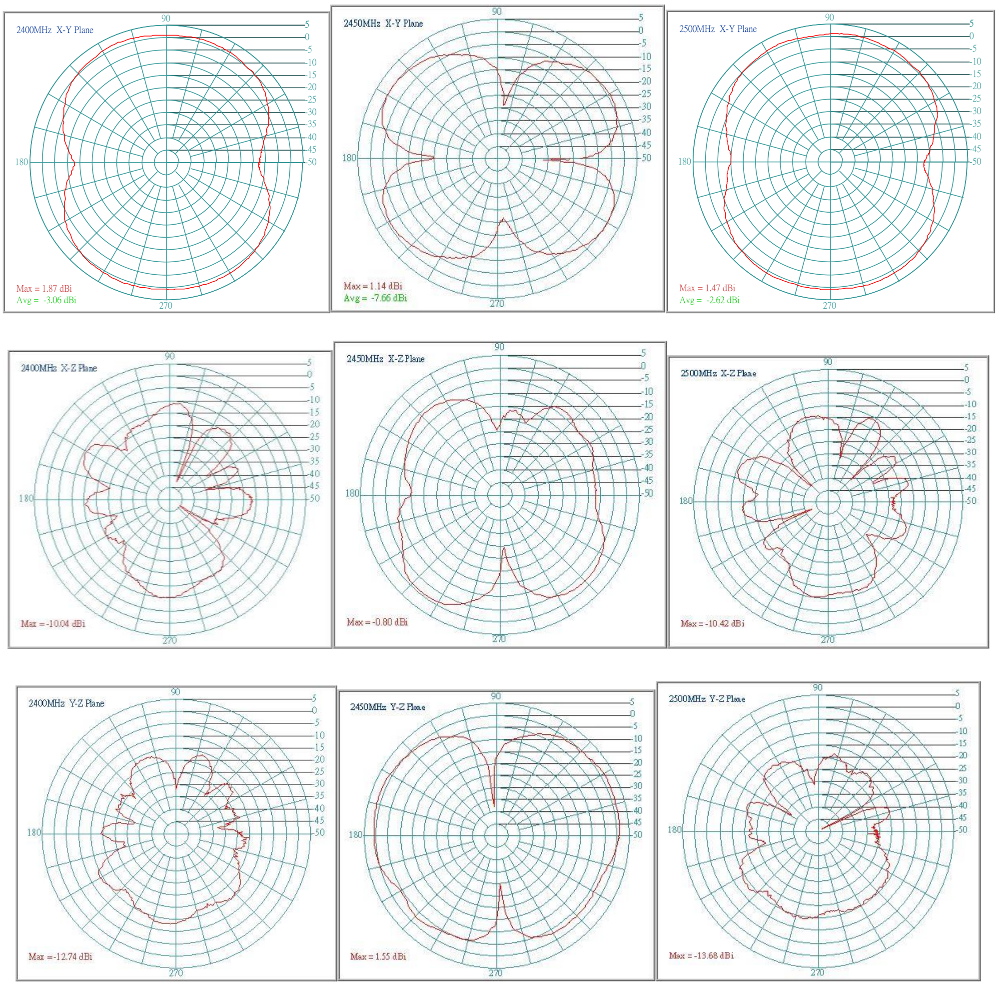
		
X-Y Plane	**X-Z Plane**	**Y-Z Plane**

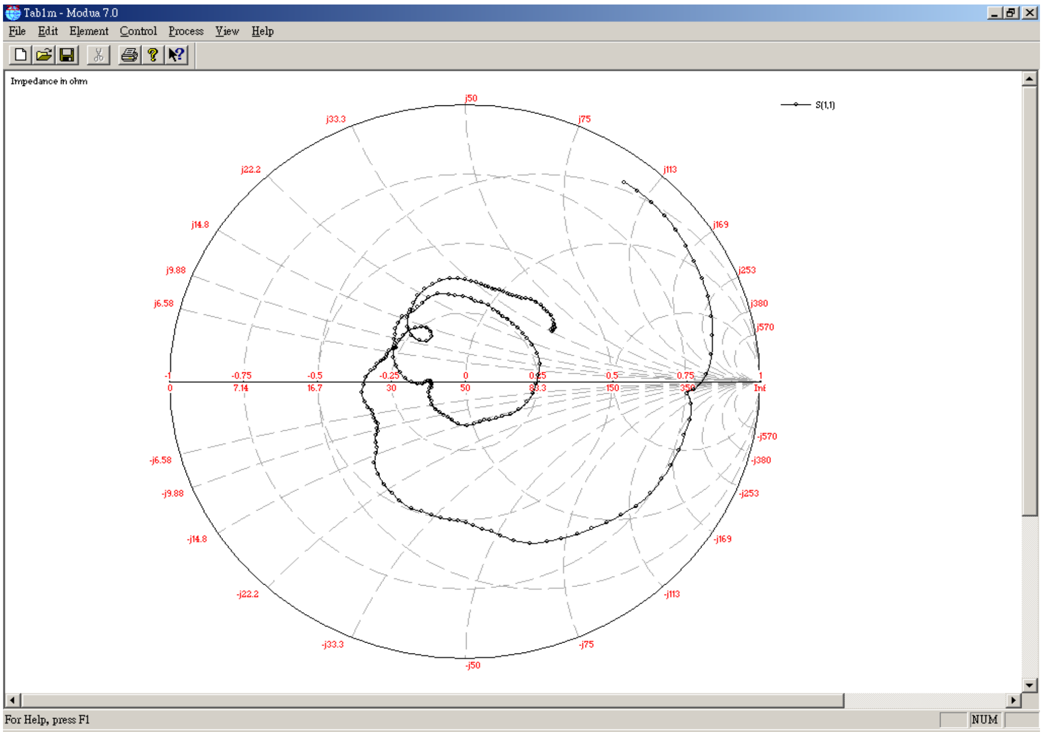
Test Result

Freq. (MHz)	X-Y Plane		X-Z Plane		Y-Z Plane	
	Vertical	Horizontal	Vertical	Horizontal	Vertical	Horizontal
2400	**1.87**	**1.71**	**-10.04**	**-0.42**	**-12.74**	**2.26**
2450	**1.66**	**1.14**	**-10.00**	**-0.80**	**-13.37**	**1.55**
2500	**1.47**	**0.88**	**-10.42**	**-0.09**	**-13.68**	**1.93**

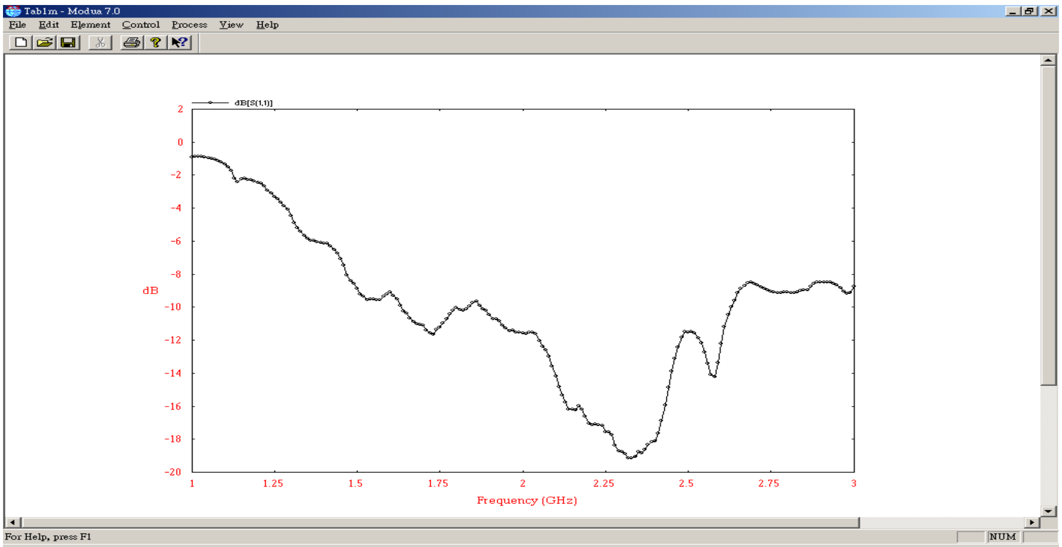
Unit：dBi

2.4G~2.5GHz 的量測結果表

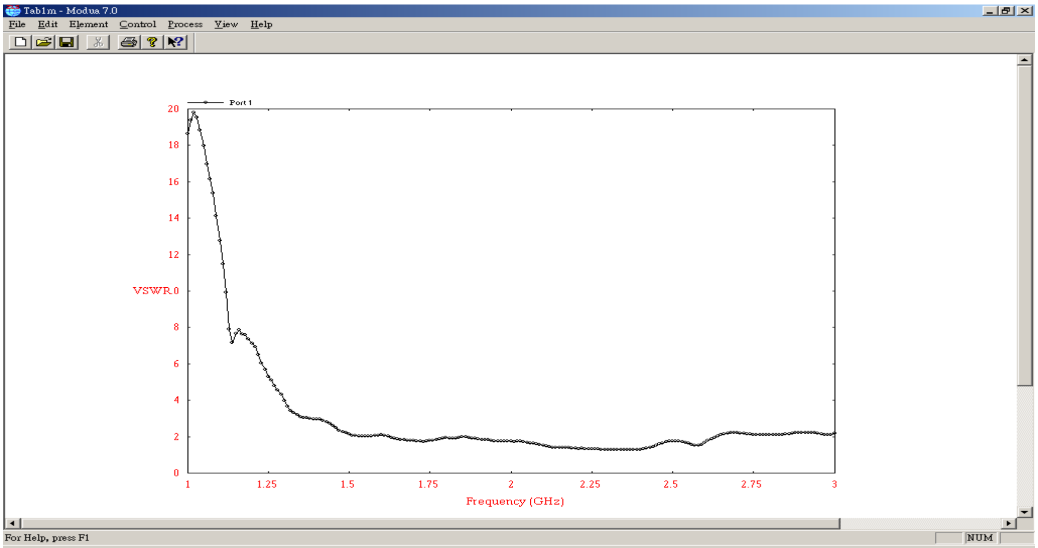




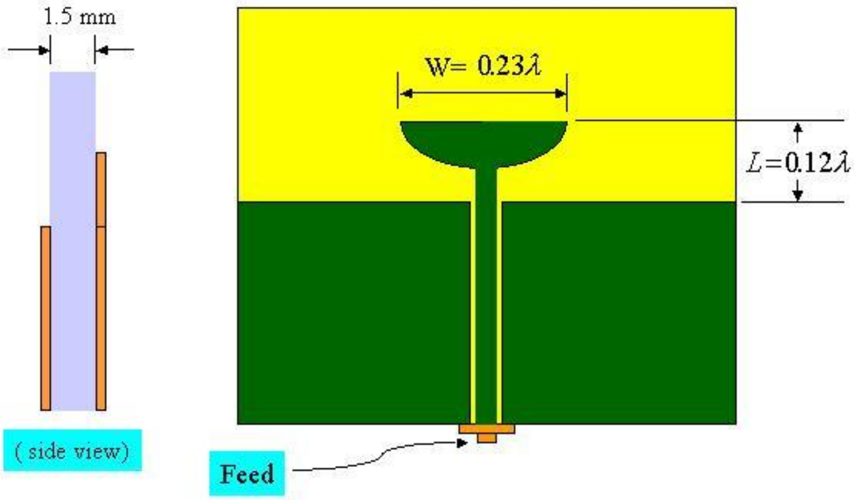
The Impedance of the Tab Monopole in the Smith Chart



The S_{11} parameter of the Tab Monopole



The VSWR of Tab Monopole



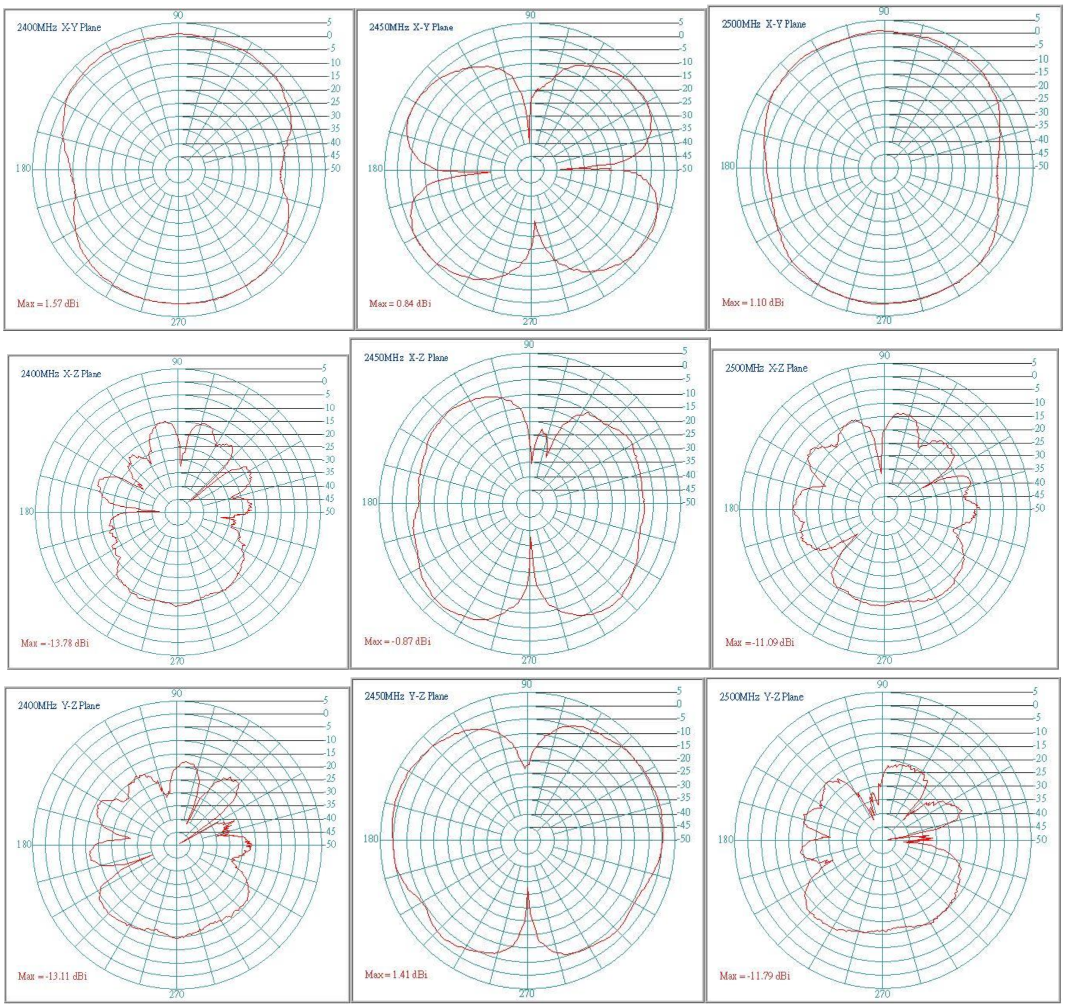
半圓型輻射體之Tab Monopole天線俯視圖

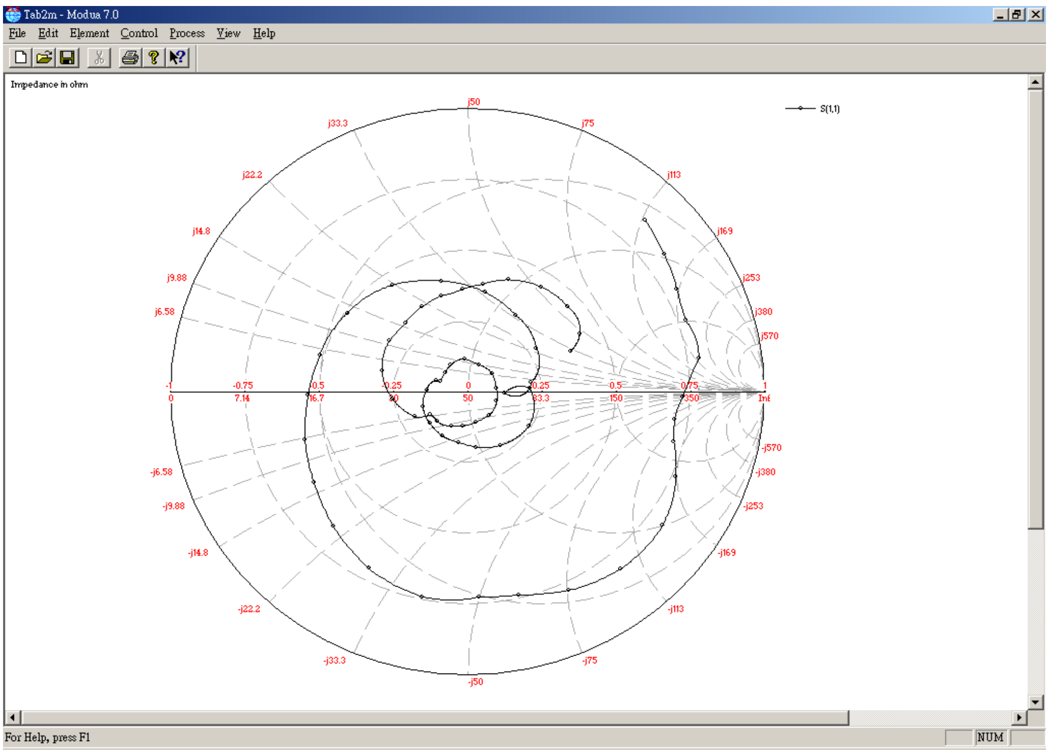
Test Result

Freq. (MHz)	X-Y Plane		X-Z Plane		Y-Z Plane	
	Vertical	**Horizontal**	**Vertical**	Horizontal	**Vertical**	Horizontal
2400	**1.57**	**1.30**	**-13.78**	**-0.76**	**-13.11**	**1.81**
2450	**1.24**	**0.84**	**-12.96**	**-0.87**	**-10.89**	**1.41**
2500	**1.10**	**1.31**	**-11.09**	**-0.43**	**-11.79**	**0.86**

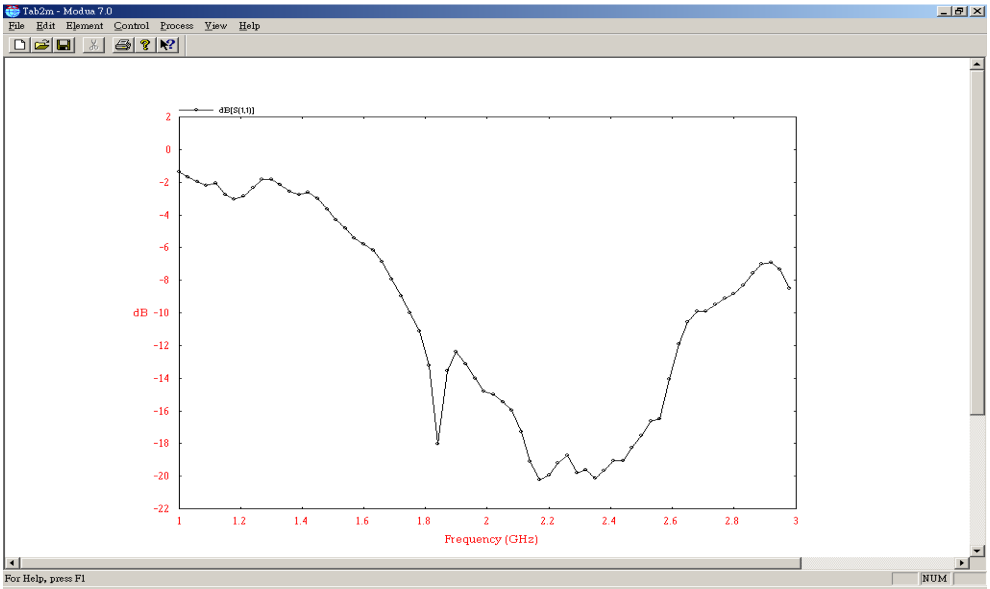
Unit：dBi

2.4G~2.5GHz 的量測結果表

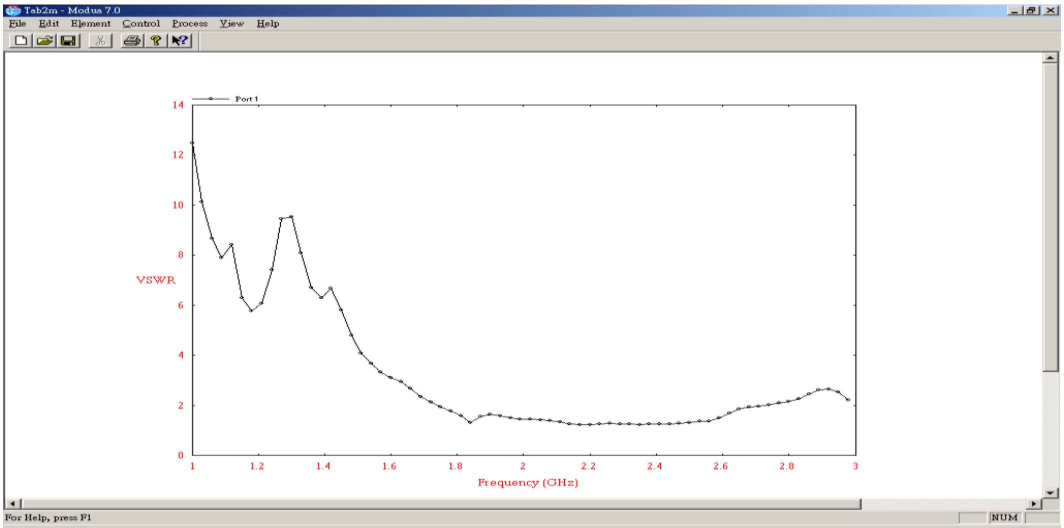




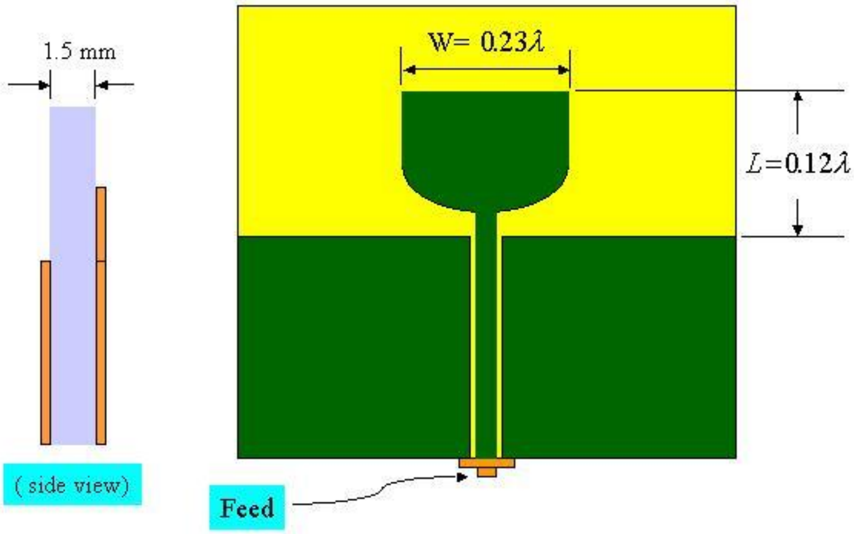
The Impedance of the Semi-Circular Tab Monopole in the Smith Chart



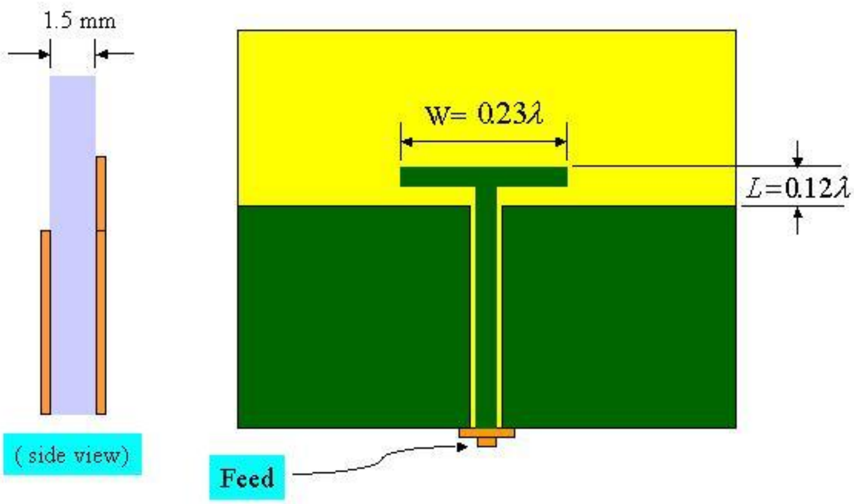
The S_{11} parameter of the Semi-Circular Tab Monopole



The VSWR of the Semi-Circular Tab Monopole

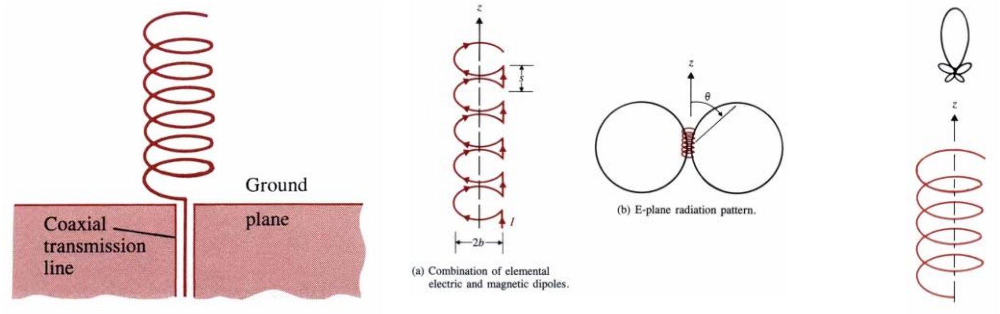


酒杯型輻射體之Tab Monopole天線俯視圖



T型輻射體之Tab Monopole天線俯視圖

8-5 Helical Antenna



Normal mode (s, $2b \ll \lambda$): Its behavior is like an electric dipole

Axial mode (s, $2b \approx \lambda$): Its mainbeam placed in the endfire direction.

$$\vec{E} = \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi = \frac{N\omega\mu_0 I}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)(\hat{a}_\theta js + \hat{a}_\phi \beta\pi b^2)\sin\theta$$: Elliptically-polarized.

If $s = \beta\pi b^2$ or $b = \frac{1}{\pi}\sqrt{\frac{s\lambda}{2}}$, it becomes circularly-polarized.

Eg. A helical antenna operating in the normal mode has N turns with diameter $2b$ and interturn spacing s. Both $2b$ and s are very small in comparison to λ / N and are adjusted to radiate circularly polarized waves. Find (a) its directive gain and directivity, (b) its radiation resistance.

(Sol.) (a) $\vec{E} = \frac{N\omega\mu_0 I}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)[\hat{a}_\theta js + \hat{a}_\varphi \beta\pi b^2]\sin\theta$, $\vec{H} = \frac{1}{\eta_0}\hat{a}_R \times \vec{E} = \frac{N\beta I}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)[\hat{a}_\phi js - \hat{a}_\theta \beta\pi b^2]\sin\theta$

Circularly polarized: $s = \beta\pi b^2$,

$$U = R^2 \hat{a}_R \cdot P_{av} = R^2 \hat{a}_R \frac{1}{2}\mathrm{Re}[\vec{E} \times \vec{H}] = \frac{\beta^2\eta_0}{16\pi^2}(NIs)^2 \sin^2\theta$$

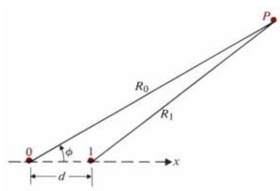
$$P_r = \int_0^{2\pi}\int_0^{\pi} U \sin\theta d\theta d\phi = \frac{\beta^2\eta_0}{b\pi}(NIs)^2 \Rightarrow G_D = \frac{4\pi U}{P_r} = \frac{3}{2}\sin^2\theta,\quad D = G_D\left(\frac{\pi}{2}\right) = 1.5$$

(b) $R_r = \frac{2P_r}{I^2} = \frac{\eta_0 (NIs)^2}{3\pi} = 40(N\beta^2\pi b^2)^2$

Note: Receiving antenna's pattern is identical with transmitting one's.

8-6 Antenna Arrays

Two-element antenna array: (In case of no coupling between antennas)



$$R_1 \cong R_0 - d\sin\theta\cos\phi$$

$$E = E_0 + E_1 = E_m F(\theta,\phi)[\frac{e^{-j\beta R_0}}{R_0} + \frac{e^{j\xi}e^{-j\beta R_1}}{R_1}]$$

$$= E_m \frac{F(\theta,\phi)}{R_0} e^{-j\beta R_0}[1 + e^{j\beta d\sin\theta\cos\phi}e^{j\xi}]$$

$$= E_m \frac{F(\theta,\phi)}{R_0} e^{-j\beta R_0} e^{j\psi/2}(2\cos\frac{\psi}{2})$$, where $\Psi=\beta d\sin\theta\cos\varphi+\xi$

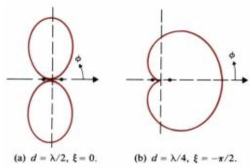
$$\Rightarrow |E| = \frac{2E_m}{R_{,0}}|F(\theta,\phi)| \cdot \left|\cos\frac{\psi}{2}\right|$$ =**Element Factor×Array Factor**

Eg. Plot the *H*-plane radiation patterns of two parallel dipoles for the following two cases: (a) $d=\lambda/2, \xi=0$, **(b)** $d=\lambda/4, \xi=-\pi/2$.

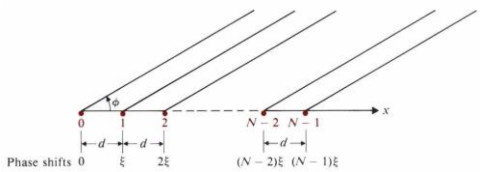
(Sol.) Let the dipole is *z*-directed

In the *H*-plane $(\theta=\pi/2)$: $|A(\phi)| = \left|\cos\frac{\psi}{2}\right| = \left|\cos\frac{1}{2}(\beta d\cos\phi+\xi)\right|$

(a) $|A(\phi)| = \left|\cos(\frac{\pi}{2}\cos\phi)\right|$, (b) $|A(\phi)| = \left|\cos\frac{\pi}{4}(\cos\phi - 1)\right|$



General Uniform Linear Arrays:



Normalized array factor in the *xy*-plane ($\theta=\pi/2$)**:**

$$|A(\Psi)| = \frac{1}{N}\left|1 + e^{j\Psi} + e^{j2\Psi} + \ldots + e^{j(N-1)\Psi}\right|$$

$$= \frac{1}{N}\left|\frac{1-e^{jN\Psi}}{1-e^{j\Psi}}\right| = \frac{1}{N}\left|\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}\right|$$, where $\Psi=\beta d\sin(\theta)\cos\varphi+\xi=\beta d\cos\varphi+\xi$ if $\theta=\pi/2$

Mainbeam direction, φ_0: ∵ Max at $\Psi=0$, ∴ $\beta d\cos\varphi_0+\xi=0 \Rightarrow \cos\phi_0 = \frac{-\xi}{\beta d}$

Null locations: $\frac{N\psi}{2} = \pm k\pi$, k=1,2,3,…

Sidelobe locations: $\frac{N\Psi}{2} = \pm(2m+1)\frac{\pi}{2}$, m=1, 2, 3, …

The first sidelobe level: $\frac{N\Psi}{2}=\pm\frac{3\pi}{2}$, $|A(\Psi)|=\frac{1}{N}\left|\frac{1}{\sin(2\pi/3N)}\right|=0.212\ \ (as\ N\to\infty)$

Broadside array $(\phi_0=\pm\frac{\pi}{2},\xi=0)$ **:** $|E_{max}|$ occurs at a direction $\perp$ the line of arrays.

Endfire array $(\phi_0=0,\xi=-\beta d)$ **:** $|E_{max}|$ occurs at a direction // the line of arrays.

Beamwidth between two first nulls: $\frac{N\Psi_1}{2}=\pi,\frac{N\Psi_2}{2}=-\pi\Rightarrow\Psi_1-\Psi_2=\frac{4\pi}{N}$

$$2\,\Delta\phi\Rightarrow(\beta d\cos\phi_1+\xi)-(\beta d\cos\phi_2+\xi)=\beta d(\cos\phi_1-\cos\phi_2)=\frac{4\pi}{N}$$

Let $\phi_1=\phi_0+\Delta\phi,\ \ \phi_2=\phi_0-\Delta\phi$

$(\phi_0=\frac{\pi}{2})\Rightarrow\Delta\phi=\sin^{-1}(\frac{\lambda}{Nd})$ for a broadside array.

$(\phi_0=0)\Rightarrow\Delta\phi\approx\sqrt{\frac{2\lambda}{Nd}}$ for an endfire array.

Eg. For a uniform linear array of 12 elements spaced $\lambda/2$ apart. Sketch the normalized array pattern $|A(\Psi)|$.

(Sol.) $d=\frac{\lambda}{2}$, $\beta d=\pi$, $|A(\Psi)|=\frac{1}{N}\left|\frac{\sin(N\Psi/2)}{\sin(\Psi/2)}\right|=\frac{1}{12}\left|\frac{\sin(6\Psi)}{\sin(\Psi/2)}\right|$

Endfire $\Rightarrow\xi=-\pi,\Psi=\beta d\cos\phi+\xi=\pi\cos\phi-\pi=\pi(\cos\phi-1)$

Broadside $\Rightarrow\xi=0,\Psi=\beta d\cos\phi+\xi=\pi\cos\phi$

Half-power point: $\frac{\sin(6\Psi)}{12\sin(\Psi/2)}=\frac{1}{\sqrt{2}}\Rightarrow 2\Delta\phi=\begin{cases}9.55(\lambda/d) & \text{degree for endfire array}\\ 46.78\sqrt{\lambda/d} & \text{degree for broadside array}\end{cases}$

Eg. Consider a five-element broadside binomial array. (a) Determine the relative excitation amplitudes in the array elements. (b) Plot the array factor for $d=\lambda/2$. (c) Determine the half-power beamwidth and compare it with that of a five-element uniform array having the same element spacings.

(Sol.) 1:4:6:4:1, broadside $\Rightarrow\xi=0$

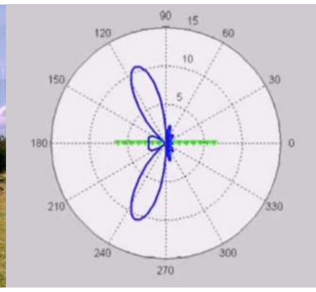
(a) $|A(\Psi)|=\frac{1}{16}\left|1+4e^{j\Psi}+6e^{j2\Psi}+4e^{j3\Psi}+e^{j4\Psi}\right|=\frac{1}{16}\left|6+8\cos\Psi+2\cos2\Psi\right|$, where $\Psi=\beta d\cos\phi+\xi$

(b) $d=\frac{\lambda}{2}$, $\beta d=\pi$, and $\xi=0$ $\Rightarrow|A(\Psi)|=\frac{1}{4}[1+\cos(\pi\cos\phi)]^2$

(c) $\frac{1}{4}[1+\cos(\pi\cos\phi)]^2=\frac{1}{\sqrt{2}}$, $\phi=74.86°$, $\therefore$ $2\Delta\phi=2(90°-74.86°)=30.28°$

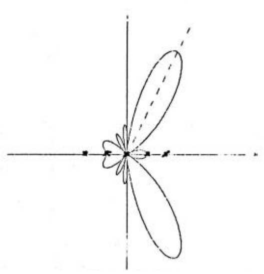
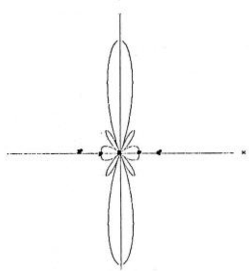
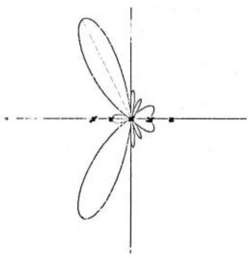
Phased Array: $\because \cos\phi_0 = \dfrac{-\xi}{\beta d}$, $\therefore$ Vary ξ electrically $\Rightarrow$ Vary φ_0 (the direction of the main beam). It can be utilized as a military radar system to scan and track a target.





Eg. Draw the far-field pattern of a phased array of dipoles with N=5, d=λ/2.

(Sol.) The effective scan range is about from $\phi_0 = 60^\circ$ to $\phi_0 = 120^\circ$ as follows.

$\xi = \dfrac{\pi}{2} \Rightarrow \phi_0 = \dfrac{\pi}{3}$ $\qquad$ $\xi = 0 \Rightarrow \phi_0 = \dfrac{\pi}{2}$ $\qquad$ $\xi = -\dfrac{\pi}{2} \Rightarrow \phi_0 = \dfrac{2\pi}{3}$

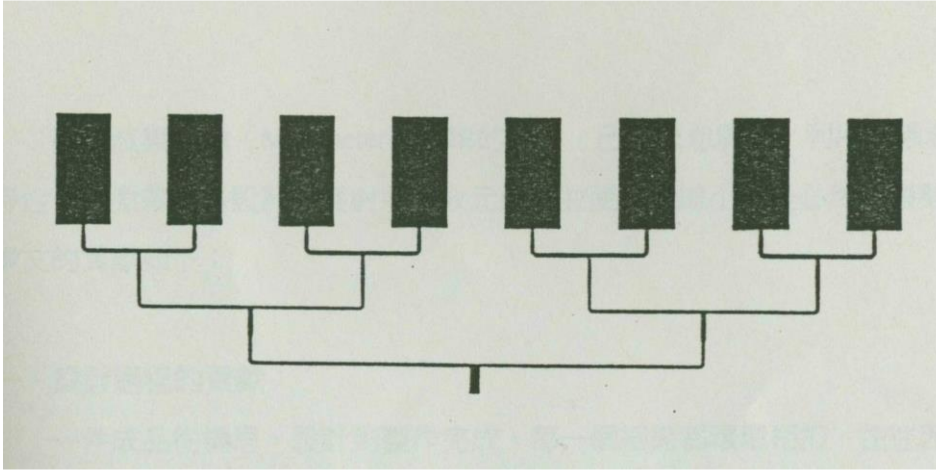
Eg. Obtain the pattern function of a uniformly excited rectangular array of $N_1 \times N_2$ parallel half-wave dipoles. Assume that the dipoles are parallel to the z-axis and their centers are spaced d_1 and d_2 apart in the x- and y-directions, respectively.

(Sol.) $F(\theta,\phi) = \left|\dfrac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right| \cdot |A_x(\Psi_x)| \cdot |A_y(\Psi_y)|$, where $|A_x(\Psi_x)| = \dfrac{1}{N_1}\left|\dfrac{\sin(\frac{N_1\Psi_x}{2})}{\sin(\frac{\Psi_x}{2})}\right|$,

$|A_y(\Psi_y)| = \dfrac{1}{N_2}\left|\dfrac{\sin(\frac{N_2\Psi_y}{2})}{\sin(\frac{\Psi_y}{2})}\right|$, $\Psi_x = \dfrac{\beta d_1}{2}\sin\theta\cos\phi + \xi_x$, and

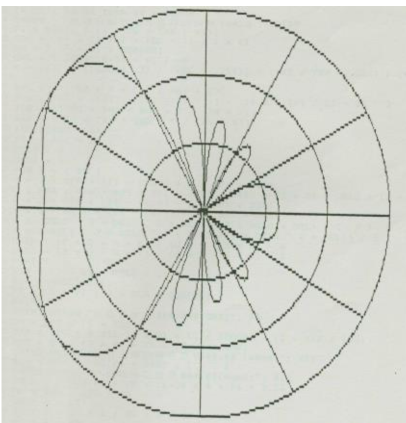
$\Psi_y = \dfrac{\beta d_2}{2}\sin\theta\cos\phi + \xi_y$

An example of microstrip linear antenna array (by Dr. I-Fong Chen)**:**

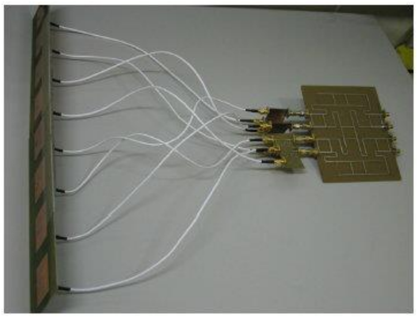


Characteristics:

型式 / 功能	FPA10000 (HEXAWAVE)	BCFPA (PROTOTYPE)
接收範圍	10.7GHZ	11.431GHZ
駐波比	2.5	2
波束 寬	65.7°	46°
增益	＞50（dB）	＞47（dB）
本地振盪頻率	10.7±0.0015（GHZ）	11.431±0.048（GHZ）
體積	54×54×6.5（cm）	12.65×4.74×0.5（cm）
特性阻抗	75Ω	50Ω
型式	Active	Passive
指向性	10.4（dB）	11.9（dB）

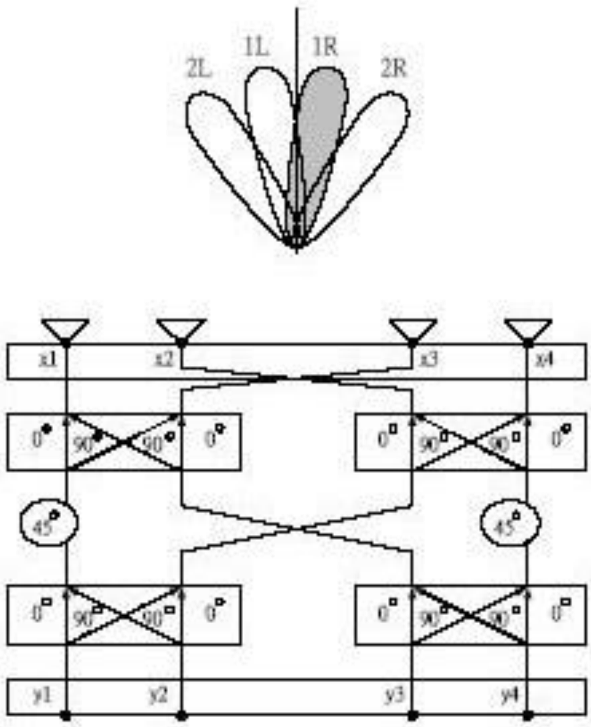


An example of smart 4-beam phased antenna array: (by W. –R. Li and Dr. K. –H. Lin)



四波束切換式陣列天線

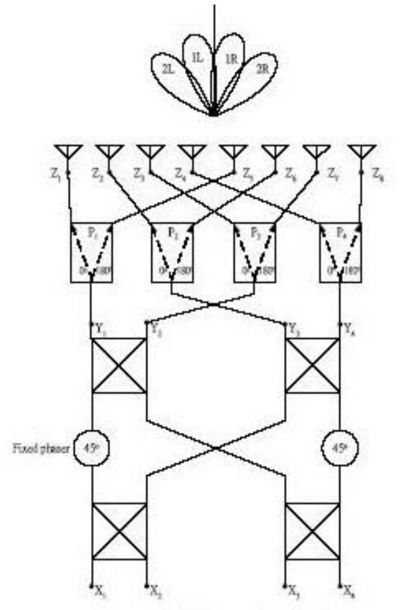
Design of *n*×*m* Butler matrix:



4×4巴特勒矩陣電路圖

表　4×4巴特勒矩陣之元素相位分布和波束方向分布表格

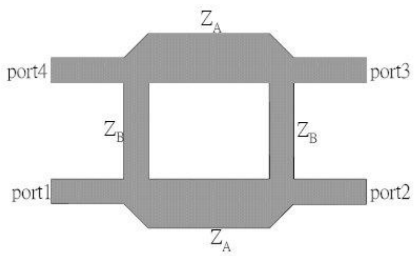
	元素 x1	元素 x2	元素 x3	元素 x4	元素間的電流相位差	波束的正向角 θ (boresight angle)
Y1	-45°	-90°	-135°	-180°	-45°	14.5° (1R)
Y2	-135°	0°	-225° (135°)	-90°	135°	-48.6° (2L)
Y3	-90°	-225° (135°)	0°	-135°	-135°	48.6° (2R)
Y4	-180°	-135°	-90°	-45°	45°	-14.5° (1L)

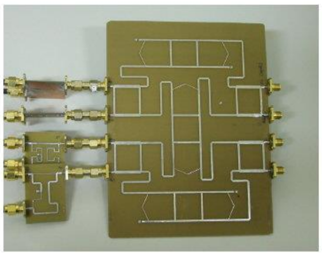


4×8 的巴特勒矩陣

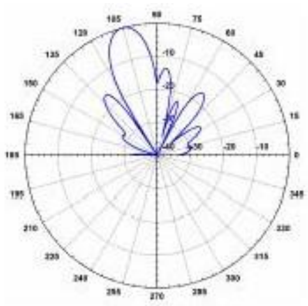
4×8巴特勒矩陣輸出端的激發電流大小

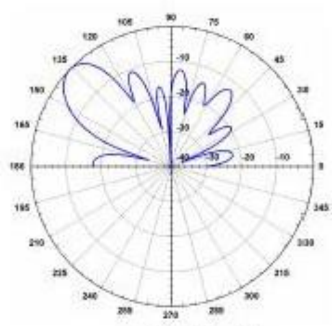
Z_1	1
Z_2	1.139
Z_3	1.509
Z_4	1.724
Z_5	1.724
Z_6	1.509
Z_7	1.139
Z_8	1

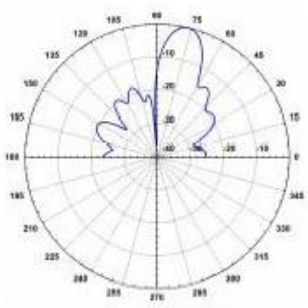


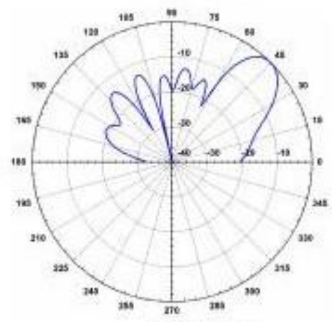

4×8 的巴特勒矩陣電路板

Patterns:

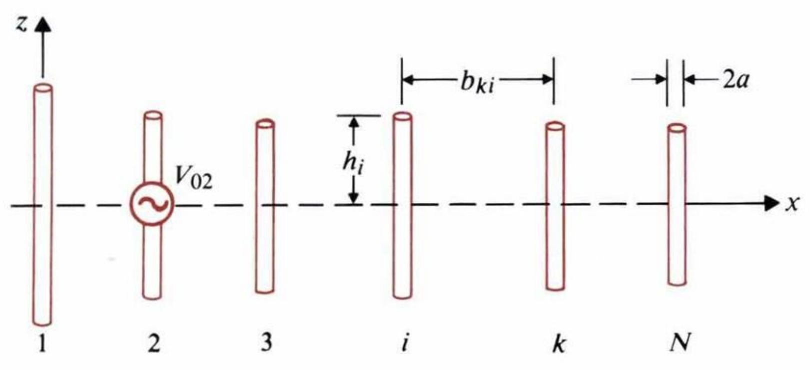

1L 波束指向 −15°


2L 波束指向 −45°


1R 波束指向 14°


2R 波束指向 44°

Yagi-Uda Antenna: A kind of endfire array.



Two types:

Antenna Dimensions

Element lengths	$2h_1$ 0.510λ	$2h_2$ 0.490λ	$2h_3 = 2h_4 = 2h_5 = 2h_6$ 0.430λ
Element spacings	b_{12} 0.250λ		$b_{23} = b_{34} = b_{45} = b_{56}$ 0.310λ

Pattern Characteristics

Directivity (Referring to λ/2 Dipole)	Half-power Beamwidth	First Sidelobes	Front-to-back Ratio
7.54 (8.77 dB)	45°	−7.2 (dB)	9.52 (dB)

The directivity of a half-wave dipole is 1.64 or 2.15 (dB)

Antenna Dimensions

Element lengths	$2h_1$ 0.476λ	$2h_2$ 0.452λ	$2h_3$ 0.436λ	$2h_4$ 0.430λ	$2h_5$ 0.434λ	$2h_6$ 0.430λ
Element spacings	b_{12} 0.250λ	b_{23} 0.289λ	b_{34} 0.406λ	b_{45} 0.323λ	b_{56} 0.422λ	

Pattern Characteristics

Directivity (Referring to λ/2-Dipole)	Half-power Beamwidth	First Sidelobes	Front-to-back Ratio
13.36 (12.58 dB)	37°	−10.9 (dB)	10.04 (dB)

8-7 Effective Areas of Antennas and Gains

Effective area: $A_e = P_L / P_{av}$ or $P_L = A_e P_{av}$

Relation between the gain and the effective area:

$$P_{av} = \frac{P_t}{4\pi r^2} G_{DA} \Rightarrow \frac{P_L}{P_t} = \frac{A_{eB} G_{DA}}{4\pi r^2}, \text{ and } \frac{G_{DA}}{A_{eA}} = \frac{G_{DB}}{A_{eB}} \text{ for all antennas}$$

Eg. Determine the effective area, $A_e(\theta)$, of an elemental electric dipole of a length dl (<<λ) used to receive an incident plane electromagnetic wave of wavelength.

(Sol.) $A_e(\theta) = \frac{P_L}{P_{av}} = \frac{\eta_0}{4R_r}(d\ell)^2 \sin^2\theta = \frac{3}{8\pi}(\lambda \sin\theta)^2$. $\because$ $G_D(\theta,\phi) = \frac{3}{2}\sin^2\theta$,

$$\therefore \frac{(3/2)\sin^2\theta}{(3/8\pi)(\lambda\sin\theta)^2} = \frac{G_D(\theta,\phi)}{A_e(\theta,\phi)} = \frac{4\pi}{\lambda^2} \Rightarrow G_D(\theta,\phi) = \frac{4\pi}{\lambda^2} A_e(\theta,\phi)$$

Under matched condition: $P_L = \frac{V_{oc}^2}{8R_r} = \frac{(-\ell_e E_i)^2}{8R_r}$ and $P_{av} = \frac{E_i^2}{2\eta_0}$

$$\Rightarrow A_e(\theta) = \frac{30\pi}{R_r} \ell_e^{\,2}(\theta)$$

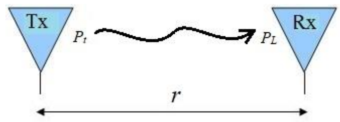
Eg. Assume that a linearly polarized plane electromagnetic wave is incident on a half-wave dipole, (a) obtain an expression for the effective area $A_e(\theta)$. (b) Calculate the maximum value of A_e for 100*MHz*.

(Sol.) (a) For a half-wave dipole, $\ell_e(\theta) = \frac{2}{\beta}\left[\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right]$

$$\Rightarrow A_e(\theta) = \frac{30\pi}{R_r}\ell_e^2(\theta) = \frac{30\pi}{73.1}\ell_e^2 = 1.28\ell_e^2(\theta) = 0.129\lambda^2\left[\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right]^2$$

(b) $\beta = \frac{2\pi \times 100 \times 10^6}{3\times 10^8} = \frac{2\pi}{3}$, $\frac{2}{\beta} = \frac{3}{\pi}$, $\frac{dA_e(\theta)}{d\theta} = 0 \Rightarrow \theta_{max} = \frac{\pi}{2}$, $A_e(\theta = \frac{\pi}{2}) = 1.17\,cm^2$

8-8 Friis Transmission Formula and Radar Equation



Friis transmission formula:

$$\frac{P_L}{P_t}=(\frac{A_{e2}}{4\pi r^2})G_{D1}=\frac{A_{e2}}{4\pi r^2}\cdot\frac{4\pi A_{e1}}{\lambda^2}=\frac{A_{e1}A_{e2}}{r^2\lambda^2}=\frac{G_{D1}G_{D2}\lambda^2}{(4\pi r)^2}$$

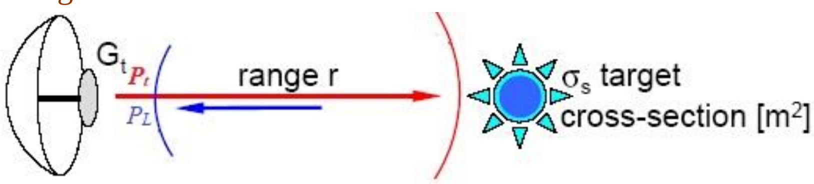
Eg. Communication is to be established between two stations 1.5*km* apart that operate at 300*MHz*. Each is equipped with a half-wave dipole. (a) If 100*W* is transmitted from one station, how much power is received by a matched load at the other station? (b) Repeat (a) assuming that both antennas are Hertzian dipoles.

(Sol.) (a) $\frac{P_L}{P_t}=\frac{G_{D1}G_{D2}\lambda^2}{16\pi^2 r^2}$. Half-wave dipole: G_D=1.64, $f=300\times10^6\Rightarrow\lambda=1m$

$P_t=100W$, $P_L=P_t\cdot\frac{1.64^2\cdot1^2}{16\pi^2\cdot(1500)^2}=7.6\times10^{-7}W=0.76\mu W$

(b) $G_D=1.5\Rightarrow P_L=6.33\times10^{-7}W=0.633\mu W$

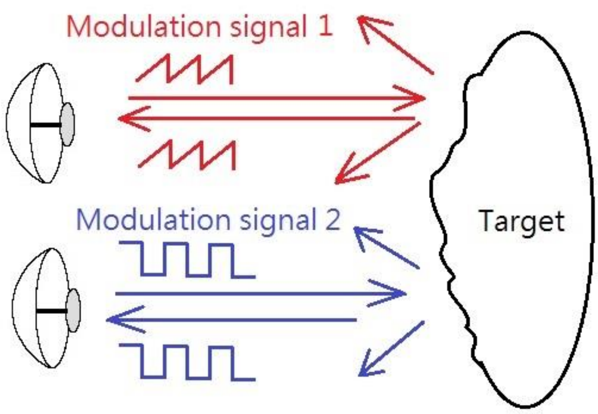
Radar equation: Radar is a transmit–receive system. Define σ_{bs}= radar cross section of target



∵ Usually, a radar has an antenna to transmit or receive waves, ∴ $G_{D1}=G_{D2}=G_D$.

Define $P_T=\frac{P_tG_D}{4\pi r^2}$ as power density at a distance r, $\frac{\sigma_{bs}P_T}{4\pi r^2}$ as power density reflected by the target $\Rightarrow P_L=A_e\cdot\frac{P_T\sigma_{bs}}{4\pi r^2}=A_e\sigma_{bs}(\frac{P_t}{(4\pi r^2)^2})G_D(\theta,\phi)$

$$\because\ A_e=\frac{\lambda^2}{4\pi}G_D(\theta,\phi),\ \therefore\ \frac{P_L}{P_t}=\frac{\sigma_{bs}\lambda^2}{(4\pi)^3r^4}G_D^2(\theta,\phi)=\frac{\sigma_{bs}}{4\pi}(\frac{A_e}{\lambda r^2})^2$$



Eg. Assuming that 50*kW* is fed into the antenna of a radar system operating at 3*GHz*. The antenna has an effective area of 4*m*^{2} and a radiation efficiency of 90%. The minimum detectable signal power (over noise inherent in the receiving system and from the environment) is 1.5*pW*, and the power reflection coefficient for the antenna on receiving is 0.05. Determine the maximum usable range of the radar for detecting a target with a backscatter cross section of 1*m*^{2}.

(Sol.) $f=3\times10^9$Hz, $\sigma_{bs}=1m^2$, $\lambda=0.1m$, $A_e=4m^2$, $P_t=0.9\times5\times10^4=4.5\times10^4W$,

$$P_L=1.5\times10^{-12}\left(\frac{1}{1-0.05}\right)=1.58\times10^{-12}W,\ \because\ r^4=\frac{\sigma_{bs}A_e^2}{4\pi\lambda^2}\left(\frac{P_t}{P_L}\right),\ \therefore\ r=4.2\times10^4\ m$$

Eg. The antenna at the earth station of a satellite communication link having a gain of 55*dB* at 14*GHz* is aimed at a geostationary satellite 36500*km* away. Assume that the antenna on the satellite has a gain of 35*dB* in transmitting the signal back toward the earth station at 12*GHz*. The minimum usable signal is 8*pW*. (a) Neglecting antenna ohmic and mismatch losses, find the minimum satellite transmitting power required. (b) Find the peak transmitting pulse power needed at the earth station in order to detect the satellite as a passive object, assuming the backscatter cross section of the satellite including its solar panels as 25*m*^{2} and the minimum detectable return pulse power to be 0.5*pW*.

(Sol.) (a) $P_t=\dfrac{(4\pi r)^2}{G_eG_s\lambda_s^{\,2}}P_L$, $\lambda_e=2.14\times10^{-2}$, $\lambda_s=2.5\times10^{-2}$, $r=3.65\times10^7m$, $P_L=8\times10^{-12}W$,

$G_e=10^{55/10}=3.16\times10^5$, $G_s=10^{35/10}=3.16\times10^3\Rightarrow P_t=2.7W$

(b) $P_t=\dfrac{4\pi}{\sigma_{bs}}\left(\dfrac{\lambda_e r^2}{A_e}\right)^2P_L$, $A_e=\dfrac{\lambda_e^2}{4\pi}G_e=15.7m^2\Rightarrow P_t=1.13\times10^9\ W$

Radar Cross Section (RCS):

Define P_i as the time-average incident power density at the object, P_s: time-average scattered power density at the receiver site, σ_{bs}: backscatter cross section, and r: distance between scatter and receiver. $\sigma_{bs}=4\pi r^2\dfrac{P_s}{P_i}$ or $P_s=\dfrac{\sigma_{bs}P_i}{4\pi r^2}$. **Note:** P_s is inversely proportional to r^2 for large r, so that σ_{bs} does not change with r.

Different airplanes have **distinct** radar cross sections. A radar system can utilize this characteristic to identify the target.

中華民國空軍軍歌

Eg. A comparison among Mig-19, Mig-21, Mig-23, Mig-25, Su-27, 中共殲十戰鬥機 (above in PRC), IDF, F-16, Mirage-2000, and new IDF (above in ROC).

Mig-19

Mig-21

Mig-23

Mig-25

Su-27

中共殲十(J-10)戰鬥機

IDF

F-16

Mirage-2000

new IDF

Stealth airplanes/vessels: (1) Specific shape to reduce RCS. (2) Coating can be utilized to absorb EM waves radiated by enemy's radar systems.

Eg. USA F-117 stealth bomber (airborne) and a comparison between F16, F22, and F35 fighters.

F-117

F22

Eg. 中共可匿蹤之殲二十(J-20)戰鬥機與傳統式殲十戰鬥機。

J-20

J-10

Eg. A comparison between Lafeya vessel and a common vessel.

Eg. 雷達未發明之前的空戰情況：我國 **814** 空戰英雄/空軍第 **4** 航空大隊大隊長高志航的電影—「寬橋英烈傳」與美國支援對日抗戰飛虎隊陳納德將軍的紀錄片。

8-9 Wave Propagation near Earth's Surface

$\vec{E} = \vec{E}_{\theta 1} + \vec{E}_{\theta 2}$, where $|\vec{E}_{\theta 1}| = k(\dfrac{e^{-j\beta R}}{R})\sin\theta$, $|\vec{E}_{\theta 2}| = \Gamma_{11}(\theta)k(\dfrac{e^{-j\beta R'}}{R'})\sin\theta'$, and

$$k = \frac{jId\ell\,\eta_0\beta}{4\pi}$$

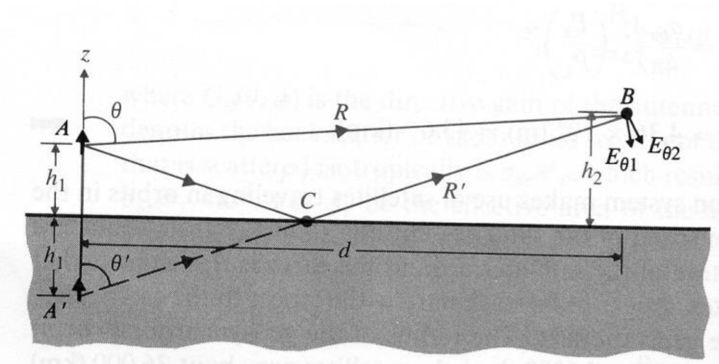
and $R' = \overline{AC} + \overline{BC} = \overline{A'B} = [d^2 + (h_2{}^2 + h_1{}^2)^2]^{1/2} \approx d + \dfrac{(h_2+h_1)^2}{2d}$,

$$R = [d^2 + (h_2 - h_1)^2]^{1/2} \approx d + \frac{(h_2-h_1)^2}{2d}, \quad \therefore R' - R = \frac{2h_1h_2}{d}$$

$$\Rightarrow \vec{E}_\theta \approx \hat{a}_\theta k(\frac{e^{-j\beta R}}{R})(\sin\theta)[1 + \Gamma_{11}(\theta)e^{-j\beta(R'-R)}]$$

$$= \hat{a}_\theta k(\frac{e^{-j\beta R}}{R})(\sin\theta)[1 + \Gamma_{11}(\theta)e^{-j\beta(h_1h_2/d)}] = \hat{a}_\theta k(\frac{e^{j\beta R}}{R})(\sin\theta)\cdot F$$

If the earth is perfect conducting, $\Gamma_{11}(\theta) = 1 \Rightarrow |F| = \left|1 - e^{-j2\beta(h_1h_2/d)}\right| = 2\left|\sin(\dfrac{2\pi h_1h_2}{\lambda d})\right|$.



Eg. A transmitting vertical half-wave dipole 60*m* above the ground radiated 400*W* at 100*MHz*. Assume the ground to be perfectly conducting. (a) Calculate the power available at a vertical half-wave receiving antenna 50*km* away at height 30*m* above the ground. (b) At a distance 50*km* from the transmitting antenna, where (at what altitudes) would there be a null field?

(Sol.) (a) $P_L = G^2(\dfrac{\lambda}{4\pi r})^2 P_t'$, $P_t' = |F|^2 P_t = \left\{2\left|\sin(\dfrac{2\pi h_1h_2}{\lambda d})\right|\right\}^2 P_t$

$$P_t = 400W,\ h_1 = 60m,\ h_2 = 30m,\ \lambda = 3m,\ d = 50000\,m \Rightarrow P'_t = 0.0225P_t \approx 9W,$$

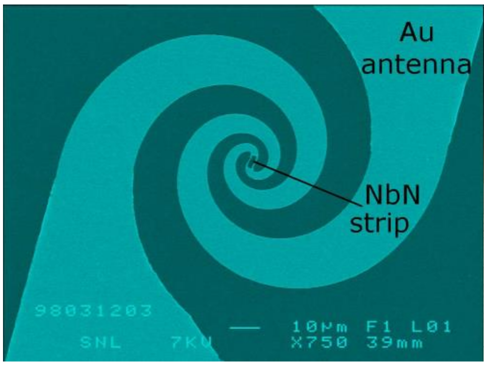
$$G{=}1.64 \Rightarrow P_L = 5.5\times10^{-10}W$$

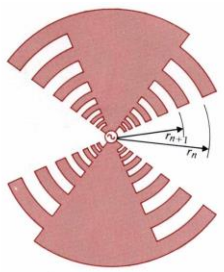
(b) Nulls: $\dfrac{2\pi h_1h_2}{\lambda d} = n\pi$, $h_1 = 60\,(m)$, $h_2 = 1.25n\,(m)$, n=1, 2, 3, ……

8-10 Broadband Antennas

Frequency-independent Antenna: The pattern and impedance characteristics are independent of frequency, because it is described entirely by angles, not dimension.

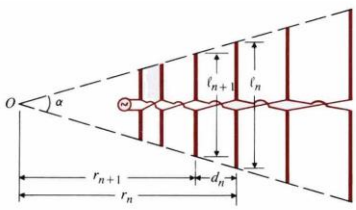
$$r = r_0 e^{a(\phi-\delta)},\quad \phi-\delta = \frac{1}{a}\ln\left(\frac{r}{r_0}\right)$$





Log-Periodic Antenna: The antenna is operated in the discrete frequency. $\dfrac{r_{n+1}}{r_n} = \dfrac{r_0 e^{a(\phi-\theta)}}{r_0 e^{a(\phi+2\pi-\theta)}} = e^{-2\pi a} = \tau$, where $f_n = \tau f_{n+1}$ or

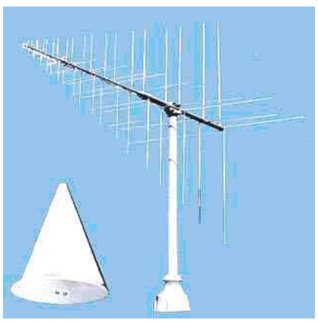
$$\ln(f_{n+1}) = \ln(f_n) + \ln\left(\frac{1}{\tau}\right)$$

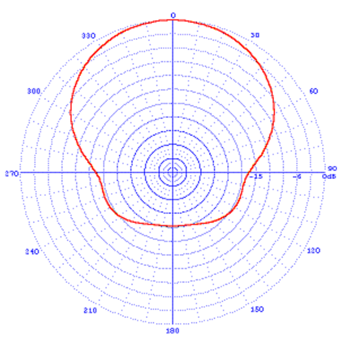


Log-periodic Dipole Antenna: $\dfrac{\ell_{n+1}}{\ell_n} = \dfrac{r_{n+1}}{r_n} = \tau$,

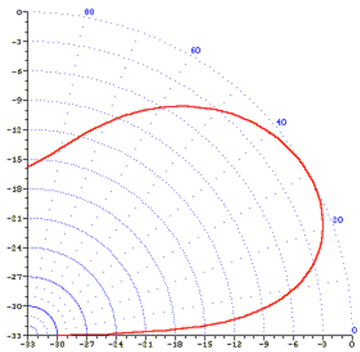
$$d_n = r_n - r_{n-1} = r_n(1-\tau) \quad \left(\text{or } \frac{d_{n+1}}{d_n} = \tau\right)$$

$$\tan\frac{\alpha}{2} = \frac{\ell_n}{2r_n} = \frac{\ell_n(1-\tau)}{2d_n} = \frac{1-\tau}{4\pi}$$





Azimuth pattern

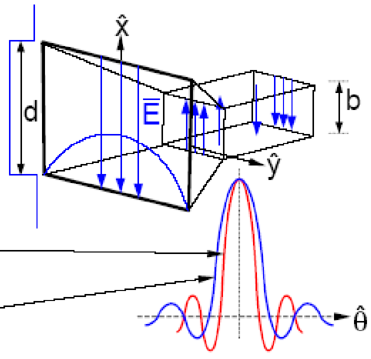


Elevation pattern

8-11 Waveguide Antennas

WAVEGUIDE ANTENNAS

Horn Antenna:



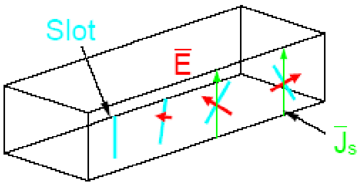
TE_{10} expands to fill aperture

Diffraction in x-plane is Fourier transform of box,
= $(\sin \theta\pi d/\lambda)/(\theta\pi d/\lambda)$

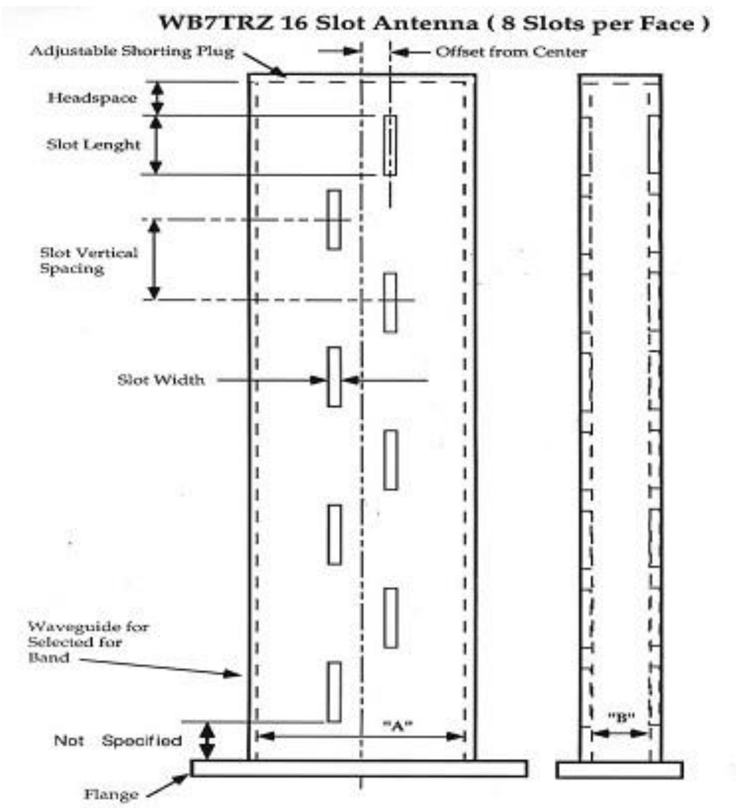
Diffraction in y plane is Fourier transform of a half sine wave (broader, but lower sidelobes)

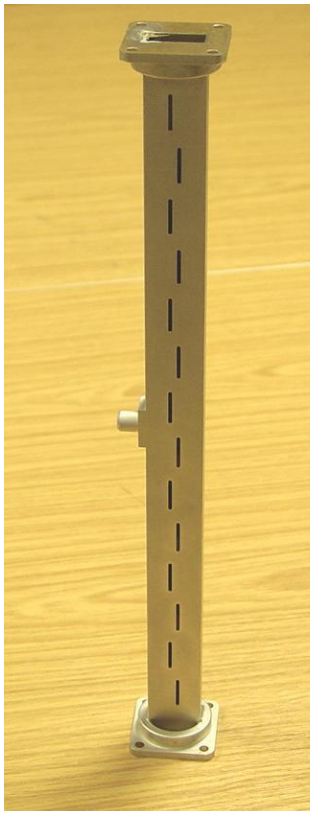
Waveguide Slot Antennas:

Tilted slots radiate in proportion to tilt and current interrupted



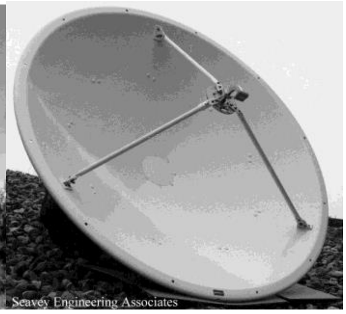
For uniform phased array, alternate tilts of alternate slots. Since $\lambda_{guide} > \lambda_o$, fill guide with ε to shorten λ_{guide} .





8-12 Reflector Antennas



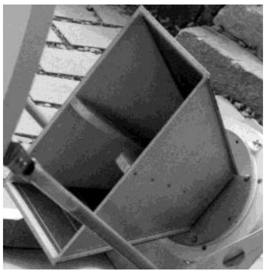


Far-field formula: $\vec{E}(R,\theta,\phi)=\dfrac{-j\omega\mu_0 e^{-jkR}}{4\pi R}\iint_{surface}(\vec{J}-(\vec{J}\cdot\hat{a}_R)\hat{a}_R)e^{jk\rho'\cdot\hat{a}_R}dS(\theta',\phi')$,

where $\vec{J}=2(\hat{a}_n\times\vec{H}_{inc})$, $\vec{H}_{inc}$ is the incident magnetic field which is radiated by the feed, $\hat{a}_n$ is the unit normal vector on the point of the reflector's surface, ρ' is the distance between the origin and the point of the reflector's surface.

In case of a circular parabolic reflector, of which focal length is f, the diameter of the aperture is D , focus is located at the origin, and the tip is at $(x,y,z)=(0,0,-f)$, then we have $\rho'(\theta',\phi')=\dfrac{2f}{1-\cos\theta'}$, $\hat{a}_n=-\hat{x}\cos\dfrac{\theta'}{2}\cos\phi'-\hat{y}\cos\dfrac{\theta'}{2}\sin\phi'+\hat{z}\sin\dfrac{\theta'}{2}$,

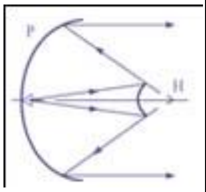
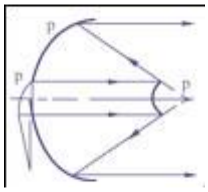
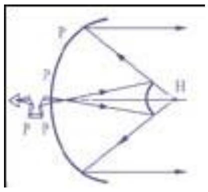
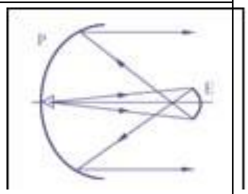
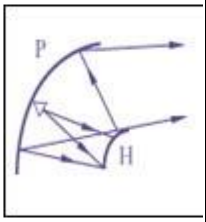
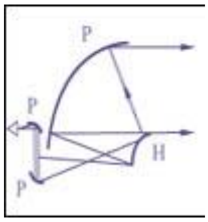
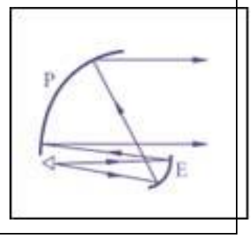
$dS(\theta',\phi')=\rho'^2\sin\theta'\csc\dfrac{\theta'}{2}d\theta' d\phi'$, $2\tan^{-1}(4f/D)\le\theta'\le\pi$, and $0\leqq\varphi'\leqq 2\pi$.



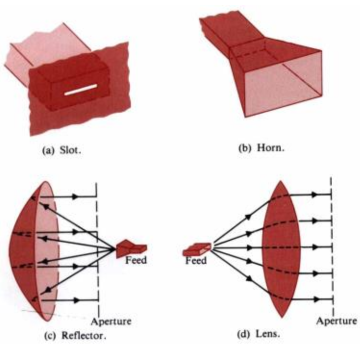
Feed of reflector antenna

Classification of dual reflector antennas:

1. Cassegrain Reflector: It is a combination of a primary concave mirror and a secondary convex mirror, both aligned symmetrically about the optical axis. The primary mirror is of paraboloid type, while the secondary mirror is of hyperboloid type.
2. Gregorian Reflector: It consists of two mirrors; the primary mirror is a concave paraboloid which collects the light and brings it to a focus before the secondary mirror. The secondary mirror is an ellipsoid.

	Cassegrain Reflector		Gregorian Reflector	
Symmetrical Type				
Non-symmetrical Type				

8-13 Aperture Antennas



$\vec{E}_a = \hat{x}E_a$, $P(R_0,\theta,\varphi)$ at the far zone ($\beta R >> 1$)

$\vec{E}_p = \hat{x}E_p$, where $\vec{E}_p = \dfrac{j}{\lambda R_0} \iint_{aperture} \vec{E}_a(x',y') e^{-j\beta R} dx'\,dy'$

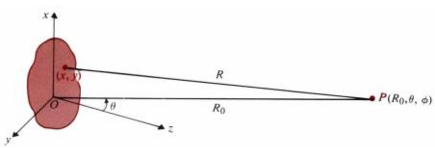
$\because \beta R >> 1,$

$\therefore \; R \approx R_0 - (\hat{x}x' + \hat{y}y') \cdot (\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi)$

$= R_0 - (x'\sin\theta\cos\phi + y'\sin\theta\sin\phi)$

$\Rightarrow \vec{E}_p = \dfrac{j}{\lambda R_0} e^{-j\beta R_0} \cdot F(\theta,\phi)$, where

$F(\theta,\varphi) = \iint_{aperture} \vec{E}_a(x',y') e^{j\beta \sin\theta(x'\cos\phi + y'\sin\phi)} dx'\,dy'$



Directivity: $D = \dfrac{4\pi U_{max}}{P_r}$,

$U_{max} = \dfrac{1}{2\eta_0} R_0^2 \left|\vec{E}_p\right|^2_{max} = \dfrac{1}{2\eta_0 \lambda^2} \left| \iint \vec{E}_a(x',y') dx'\,dy' \right|^2$

$$P_r = \frac{1}{2\eta_0} \iint \left|\vec{E}_a(x',y')\right|^2 dx'\,dy' \Rightarrow D = \frac{4\pi}{\lambda^2} \frac{\left| \iint_{aperture} \vec{E}_a(x',y') dx'\,dy' \right|^2}{\iint_{aperture} \left|\vec{E}_a(x',y')\right|^2 dx'\,dy'}$$

If $E_a(x',y')$ is constant $\Rightarrow D = 4\pi/\lambda^2$

Case 1 $\varphi=0$ (in the *xz*-plane) *Case 2* $\varphi=\pi/2$ (in the *yz*-plane)

Eg. Assume that the field in an $a\times b$ rectangular aperture in an *xy*-plane is linearly polarized in the *y*-direction and that the aperture excitation has a uniform phase and a triangular amplitude distribution $f(x)=1-\left|\frac{2}{a}x\right|$, $|x|\le\frac{a}{2}$. Find (a) the pattern function in the *xz*-plane, (b) the half-power beamwidth, (c) the location of the first nulls, and (d) the level of the first sidelobes.

(Sol.) (a) The *xz*-plane $\Rightarrow \phi=0$, $\cos\phi=1$, $\sin\phi=0$, $E_a(x',y')=f(x)\cdot 1$

$$F(\theta,\phi)=\iint_{aperture} E(x',y')\cdot e^{j\beta\sin\theta(x'\cos\phi+y'\sin\phi)}dx'dy',$$

$$F(\theta,\phi=0)=\int_{-a/2}^{a/2}[1-\left|\frac{2}{a}x\right|]e^{j\beta x'\sin\theta}dx'\cdot\int_{-b/2}^{b/2}1\cdot dy'=\frac{ab}{2}\cdot\frac{\sin^2(\frac{\beta a}{\psi}\sin\theta)}{(\frac{\beta a}{\psi}\sin\theta)^2}=F_{xz}(\theta)$$

(b) $$\frac{\sin^2(\frac{\beta a}{\psi}\sin\theta)}{(\frac{\beta a}{\psi}\sin\theta)^2}=\frac{1}{\sqrt{2}}\Rightarrow 2\Delta\theta\approx 2\times 0.326\frac{\pi\lambda}{a}=0.652\times\frac{\lambda}{a}$$

(c) $$\sin(\frac{\beta a\sin\theta}{\psi})=0\Rightarrow\theta_{null}=\sin^{-1}(\frac{2\lambda}{a})$$

(d) $$\frac{d}{d\psi}(\frac{\sin^2\psi}{\psi^2})=0\Rightarrow\psi=\frac{3\pi}{2}\Rightarrow\frac{\sin^2\psi}{\psi^2}=0.045$$

$\therefore$ The first sidelobe level = $20\log_{10}(\frac{1}{0.045})=26.9dB$

Eg. A linearly polarized uniform electric field $\vec{E}_a=\hat{x}E_0$ exists in a circular aperture of radius *b* in a conducting plane at *z*=0. Assuming *b* to be large in comparison to wavelength, (a) find an expression for the far-zone electric field, and (b) determine the width of the main beam between first nulls.

(Sol.) (a) $x'=\rho'\cos\phi'$, $y'=\rho'\sin\phi'$, and

$$x'\cos\phi+y'\sin\phi=\rho'(\cos\phi\cos\phi'+\sin\phi\sin\phi')=\rho'\cos(\phi-\phi').$$

$$F(\theta,\phi)=E_0\int_0^b\int_0^{2\pi}e^{j\beta\rho'\sin\theta\cos(\phi-\phi')}\rho'd\phi'd\rho'=E_0\int_0^b 2\pi J_0(\beta\rho'\sin\theta)\rho'd\rho'=E_0 2\pi b^2[\frac{J_1(\beta b\sin\theta)}{\beta b\sin\theta}]$$

$$\vec{E}_p=\hat{a}_x jE_0\frac{2\pi b^2}{\lambda R_0}e^{-j\beta R_0}[\frac{J_1(u)}{u}],$$ where $u=\beta b\sin\theta=\frac{2\pi b}{\lambda}\sin\theta$.

(b) The first null of the radiation pattern occurs at the first zero of $J_1(u)$: u_{11}=3.832

$\theta_1=\sin^{-1}(\frac{3.832\lambda}{2\pi b})\cong\frac{3.832\lambda}{2\pi b}=1.22\frac{\lambda}{D}(rad)$, where *D*=2*b*. The width of the main beam between the first nulls is $2\theta_1=2.44\lambda/D(rad)$.

Formulae: $\int_0^{2\pi}e^{jw\cos\phi'}d\phi'=2\pi J_0(w)$ and $\int wJ_0(w)dw=wJ_1(w)$