Chapter 2 Waves and Particles

De Broglie wavelength: $\lambda=h/p$, where p is the momentum. If it is particle's momentum, then $p=mv=m_0v/\sqrt{1-v^2/c^2}$ and $\lambda= h(\sqrt{1-v^2/c^2})/m_0v$; else if it is photon's momentum, then $p=E/c=h\nu/c$, where $h=6.626\times10^{-34}$ $(J\cdot\text{sec})=4.136\times10^{-15}$ $(\text{eV}\cdot\text{sec})$ is Plank's constant.

Eg. Find the de Broglie wavelengths of (a) a 46*g* golf ball with a velocity of 30*m*/sec, (b) an electron with a velocity of 10^7*m*/sec

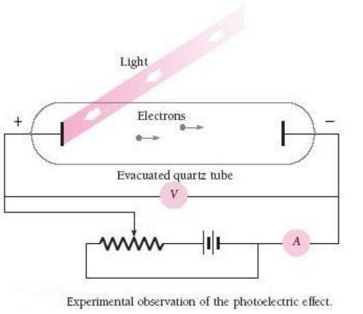
(Sol.) (a) $\because$ $v<<c$, $\lambda=h/p=h/m_0v=6.626\times10^{-34}/(46\times10^{-3}\times30)=4.8\times10^{-34}(m)$

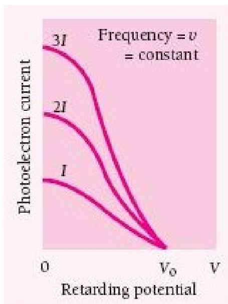
(b) $v=10^7m/\text{sec}$, $\lambda=h/p=h(\sqrt{1-v^2/c^2})/m_0v=7.3\times10^{-11}(m)$

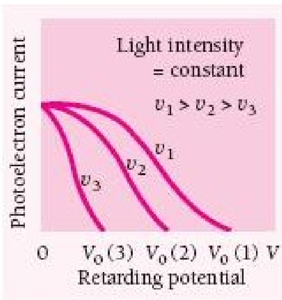
Quantum energy of one photon: $E=h\nu=hc/\lambda=1.24\times10^{-6}/\lambda=pc$.

Static mass of one photon: $m_0=h\nu/c^2$.

2-1 Photoelectric Effect $E=h\nu=K_{max}+h\nu_0$







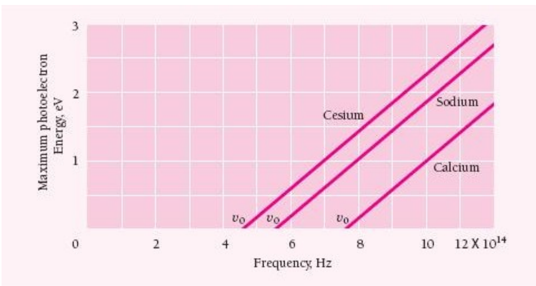


Table 2.1 Photoelectric Work Functions

Metal	Symbol	Work Function, eV
Cesium	Cs	1.9
Potassium	K	2.2
Sodium	Na	2.3
Lithium	Li	2.5
Calcium	Ca	3.2
Copper	Cu	4.7
Silver	Ag	4.7
Platinum	Pt	6.4

Photoelectric Effect: $E=h\nu=K_{max}+h\nu_0$=Maximal electron's energy + Work function

Eg. An UV light of wavelength 350*nm* and intensity 1*W*/*m*^{2} is incident at the potassium surface. (a) Find the maximum kinetic energy of photoelectrons. (b) If 0.5 percentages of the incident photons produce photoelectrons, how many photoelectrons/sec are emitted if potassium surface has an area of 1*cm*^{2}?

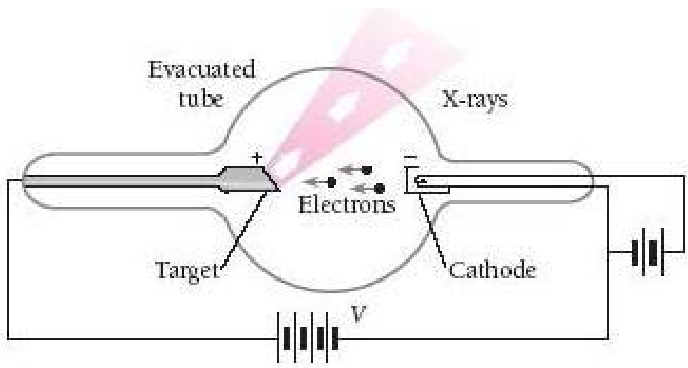
(Sol.) (a) Work function of $K=2.2\text{eV}=h\nu_0$. $E=1.24\times10^{-6}/350\times10^{-9}=3.5\text{eV}$,

$K_{max}=E-h\nu_0=3.5-2.2=1.3\text{eV}$

(b) $3.5\text{e}V=3.5\times1.602\times10^{-19}=5.68\times10^{-19}J$, $n_p=\dfrac{1\times10^{-4}}{5.68\times10^{-19}}=1.76\times10^{14}$photons/sec

$1.76\times10^{14}\times0.5\%=8.8\times10^{11}$photoelectrons/sec

Eg. Blue light of wavelength 300*nm* and intensity 0.5*W*/*m*^{2} is directed at a material with work function of 2.7*eV*. (1) Find the maximum kinetic energy of photoelectrons. (2) If the quantum efficiency is 1%, how many photoelectrons are emitted per second if the semiconductor surface has an area of 2*cm*^{2}? [台大電研]



An x-ray tube. The higher the accelerating voltage *V*, the faster the electrons and the shorter the wavelengths of the x-rays.

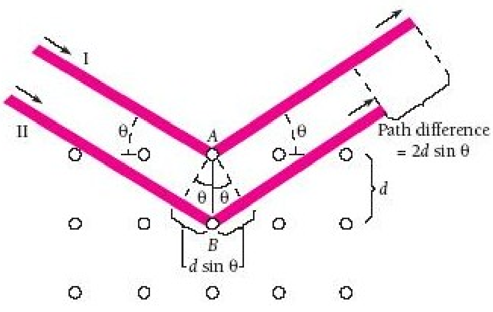
The shortest wavelength of *X*-ray due to electron bombard: $\lambda_{min}=1.24\times10^{-6}/V$, where *V* is the accelerating voltage.

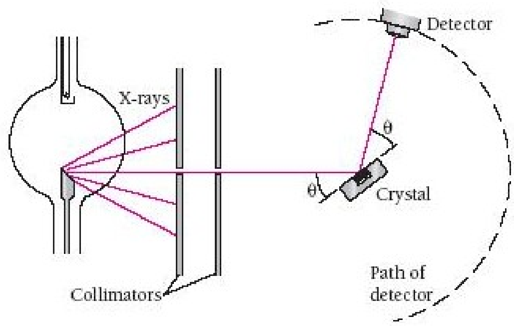
Eg. Find the shortest wavelength present in the radiation from an *X*-ray machine whose accelerating potential is 50000*V*.

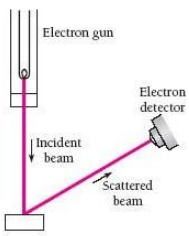
(Sol.) $\lambda_{min}=1.24\times10^{-6}/50000=2.5\times10^{-11}m=0.025nm$

2-2 Bragg's Reflection $2d\sin\theta=n\lambda$ and Compton Effect $\triangle\lambda=h(1-\cos\varphi)/m_0c$

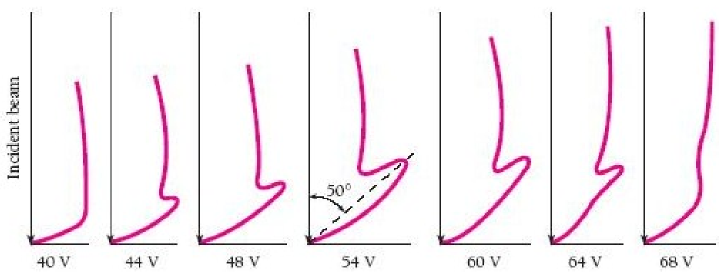
Bragg's Reflection: When $2d\sin\theta=n\lambda$ occurs, it forms a constructive interference.







The Davisson-Germer experiment.



Eg. The distance between adjacent atomic planes in calcite is 0.3*nm*. What is the smallest angle between these planes and an incident beam of 30*pm* *X*-rays at which scattered *X*-rays can be detected?

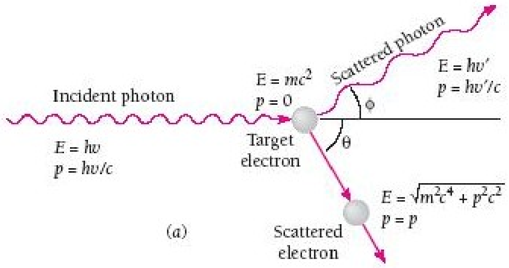
(Sol.) $d=0.3\times10^{-9}$, $\lambda=30\times10^{-12}$, $2d\sin\theta=n\lambda=\lambda\Rightarrow\sin\theta=0.05$, $\theta=2.86°$

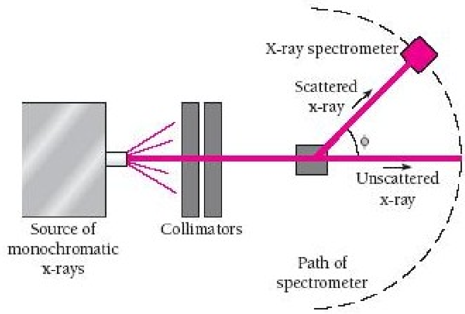
Compton Effect: $\triangle\lambda=h(1-\cos\varphi)/m_0c$

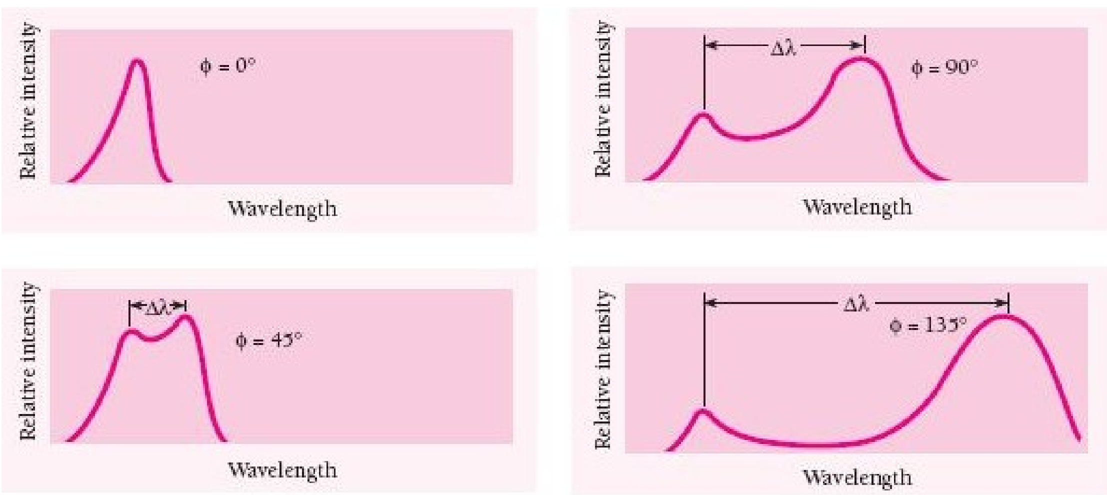
$$\begin{cases} h\nu/c+0=h\nu'\cos\phi/c+p\cos\theta \\ 0=h\nu'\sin\phi/c-p\sin\theta \end{cases}\Rightarrow(pc)^2=(h\nu)^2-2h\nu\cdot h\nu'\cos\varphi+(h\nu')^2\ldots(1)$$

And $E=K+m_0c^2=(h\nu-h\nu')+m_0c^2\Rightarrow(pc)^2=(h\nu)^2-2h\nu\cdot h\nu'+(h\nu')^2+2m_0c^2(h\nu-h\nu')\ \ldots(2)$

(1), (2)$\Rightarrow(m_0c/h)(\nu/c-\nu'/c)=(\nu/c)(\nu'/c)(1-\cos\varphi)\Rightarrow\lambda'-\lambda=\triangle\lambda=h(1-\cos\varphi)/m_0c$





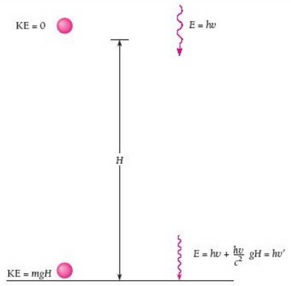


Compton wavelength: $\lambda_c=h/m_0c$, and the Compton wavelength of the electron is $\lambda_{c,electron}=2.426\times10^{-12}m$

Eg. A beam of *X*-rays is scattered by a target. At 45° from the beam direction the scattered *X*-rays have a wavelength of 2.2*pm*. What is the wavelength of the *X*-rays in the direct beam?

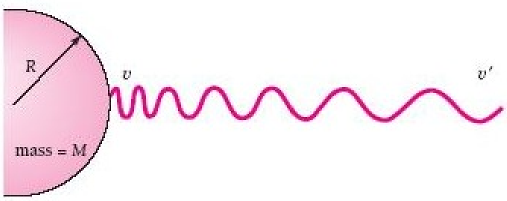
(Sol.) $\varphi=45°$, $\lambda'=2.2pm$, $h/m_0c=2.426\times10^{-12}$, $\lambda'-\lambda=h(1-\cos\varphi)/m_0c\Rightarrow\lambda=1.54pm$

2-3 Photon, Gravity, and Black Holes



Eg. Find the change in frequency of a photon of red light whose original frequency is 7.3×10^{14}*Hz* when it fall through 22.5*m*.

(Sol.) H=22.5, $\triangle\nu = gH\nu/c^2$=1.8*Hz*



Gravitational red shift: The frequency of a photon emitted from the surface of a star decreases as it moves away from the star. And its total energy becomes $E=h\nu-GMm/R=h\nu-GM(h\nu/c^2)/R=h\nu'$

$\Rightarrow \triangle\nu/\nu=(\nu-\nu')/\nu==(1-\nu'/\nu)=\dfrac{GM}{c^2R}$

(1) For most stars, M/R is small, $\triangle\nu/\nu$ is not apparent.

(2) For white dwarf stars, M/R is large, $\triangle\nu/\nu$ is measurable.

(3) For black holes, if $\dfrac{GM}{c^2R}\geqq 0.5$, we can obtain Schwarzschild radius $R_s=2GM/c^2$, and prove that no photon can leave the black holes.