



電波領域的華人大師



L. J. Chu (朱蘭成)：朱蘭成院士 1913 年 8 月 24 日生於江蘇淮陰，1934 年畢業於上海交通大學，1935 與 1938 年獲美國麻省理工學院碩士和博士學位，畢業後在麻省理工學院任教。1958 年當選第二屆中研院院士，是電磁波及雷達研究方面的三大國際權威之一，台大電機 1 館是由朱蘭成號召募資興建。1973 年逝世。

Stratton-Chu formulas for calculating EM Fields of antennas: (by L. J. Chu)

$$\vec{E}(\vec{r}) = \iiint_{V'} [-j\omega\mu\vec{J} + \frac{\rho}{\epsilon}\nabla'G - \vec{J}_m \times \nabla'G]dV' + \iint_{S'} [-j\omega\mu G(\hat{a}_n \times \vec{H}) + (\hat{a}_n \cdot \vec{E})\nabla'G + (\hat{a}_n \times \vec{E}) \times \nabla'G]dS'$$

$$\vec{H}(\vec{r}) = \iiint_{V'} [-j\omega\epsilon\vec{J}_m + \frac{\rho_m}{\mu}\nabla'G + \vec{J} \times \nabla'G]dV' + \iint_{S'} [j\omega\epsilon G(\hat{a}_n \times \vec{E}) + (\hat{a}_n \cdot \vec{H})\nabla'G + (\hat{a}_n \times \vec{H}) \times \nabla'G]dS'$$

where $G = \frac{e^{-jkr}}{4\pi r}$ is Green's function in the free space.



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我國赴北極第一人
馮簡博士訪問記
黨毅撰述 光澤地磁變化
老科學家將在風波中
在北極經歷許多有海故事
看北極光
聽馮簡博士
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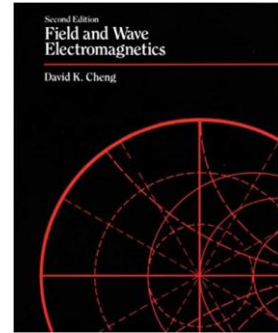


馮簡：1897 年農曆 4 月 2 日出生於江蘇嘉定，1919 年自南洋大學畢業，1921 年獲康乃爾大學碩士

學位，1924 年回國。1928 年，國民革命軍北伐成功，馮教授曾協助建立總司令部的短波通信系統。1928 年冬，馮教授應聘東北大學電氣系，並成立電波研究所。1930 年春，中央廣播事業管理處聘請馮教授籌建「中央廣播電台」。1936 年，政府邀請馮教授籌設重慶國際廣播電台，珍珠港事變後，重慶電台成為盟軍在遠東唯一可利用的短波電台，國外記者都利用這個電台轉播、傳真、發稿。抗戰末期，馮教授曾利用電台進行空中導航，使盟國機群得以在夜間遠征轟炸東北鞍山鋼廠。抗戰勝利後，馮教授並先後奉派參與相關的接收及重建工作。1949 年，京滬相繼陷共，馮教授仍固守重慶電台。當時在台北的張道藩、董顯光等先生電請先總統 蔣中正先生予以救援。馮教授及其家人遂獲安排，於同年 11 月 29 日，重慶撤守的前一天，搭乘最後一架政府的專機飛抵台灣。1950 年，馮教授受聘為台大電機系教授。初到台大時，馮教授在最短期間內恢復了電離層的觀測和研究工作。1951 年，交通部電信總局成立交通部電波研究所，聘馮教授兼任所長。1958 年八二三炮戰爆發，馮教授的學生輩馬志欽教授在金門協助國軍建立戰時通訊與天線系統，使得台灣本島與金門、小金門之間的聯繫免於中斷。1962 年張其昀先生籌建中國文化學院，延請馮教授規劃電化視聽工程系。馮教授於 1962 年 5 月 26 日晚十一時因心臟病發過世。



D. K. Cheng (鄭鈞)：1917 年出生在江蘇省，1938 年畢業於上海國立交通大學電機系，畢業後五年在中央廣播電台工作，1944 年和 1946 年分別拿到哈佛大學碩士和博士學位。1946 年在美國 USCF 劍橋研究實驗室擔任電子工程師，1948 年擔任雪城大學電機與計算機工程教授。他專攻電磁學領域，



也是該領域教科書作者。1983 年其著作 *Field and Wave Electromagnetics* 已經被 2000 多本出版物引用，2016 年全世界共有 500 多家圖書館有收藏其著作。2012 年在美國逝世。

Y. T. Lo (羅遠梓/羅遠祉)：1920 年 1 月 31 日出生於武漢，中國浙江杭州人。1942



年畢業於昆明國立西南聯合大學電機工程系（原清華大學電機工程系），獲清華大學工學士學位。1949 年獲美國伊利諾伊大學香檳分校(University of Illinois at Urbana-Champaign)電機系碩士學位。1952 年獲美國伊利諾伊大學香檳分校電機系博士學位。1952 年至 1956 年在位於美國紐約州 Ellenville 鎮的 Channel Master Corp. 公司任工程師，主持研製並設計該廠第一代電視天線。

1956 年任美國伊利諾伊大學香檳分校電機系助理教授。1956 年至 1990 年美國伊利諾伊大學香檳分校電機系助理教授、副教授、教授，電子與計算機系系主任。1969 年當選 IEEE Fellow，1986 年選為 IEEE Life Fellow，1986 年以其對天線理論和設計的創新與發明，當選美國國家工程院院士。他所提出的 microstrip patch antennas 之理論被用在設計 GPS 系統中。2002 年 5 月 10 日在美國去世。

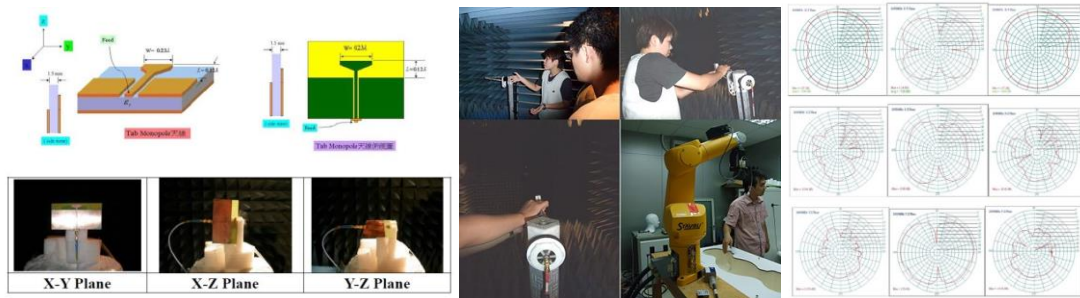
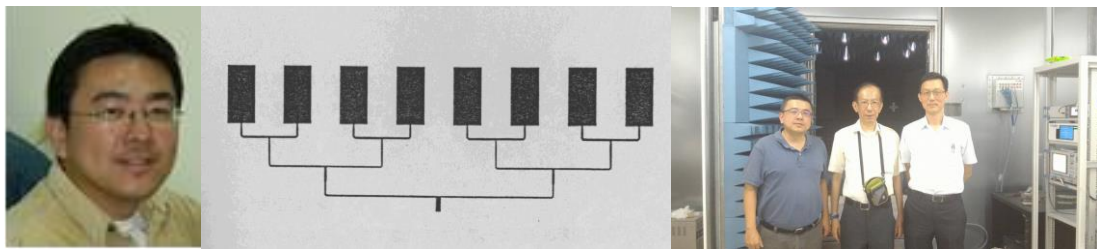
K. C. Yeh (葉公節)：1930 年生於浙江溫州，少年成長於上海，而後輾轉到台灣完成高中學業，台大電機系肄業，隨後赴美深造，於伊利諾大學香檳分校畢業後，1958 年獲史丹福大學電機博士學位。當時正逢人造衛星蓬勃發展時期，他回母校伊利諾大學展開長達 40 年的電波傳播學術教育生涯。1959 年葉教授當時的研究方向是利用當時前蘇聯的第一顆人造衛星 Sputnik 的信號探測電離層不均勻體的結構和運動特徵，之後他倡導發起和參與了眾多利用全球各地觀測台站組網探測電離層的活動，擔任眾多學術性職務。葉教授於 1981 年中山大學創校時接任電機系主任一職，在相關師資非常缺乏的情況下極力奔走聘任專業師資，一年後返回伊利諾大學。1992 年在第三任校長林基源邀請下，再度回中山並接任工學院院長。他曾主辦台灣第一屆無線電科學研討會、國際性日全蝕觀測數據的 Workshop、及領導跨國性電腦斷層的電離層觀測、研究台灣緯度特有的赤道異常區電離層等。葉前院長於 1997 年 8 月退休，返回伊利諾，於 2003 年 6 月 27 日病逝於美國，享年 73 歲。



J. A. Kong (孔金甌)：孔教授 1942 年出生於中國江蘇，2008 年去世。孔教授是孔子第 74 代孫，為台大電機學士，交大碩士，美國 Syracuse 大學博士，他自 1969 年任教麻省理工學院電機系，為電磁波泰斗，曾出版 30 本電磁學著作和 700 篇研究論文。他曾參與阿波羅十七登月計劃及探測月球表面與內部物質的設計研究；解決了雙子星電視干擾問題，發展和拓寬了微波遙測理論模式。

文大電機系在電波領域的系友名人

陳一鋒教授：1993 年文大電機系學士、1995 年中華大學電子碩士、2002 年台科大電子所博士。其獲得博士時之道賀詩：「一對天線作波心，鋒芒共振在高頻，活用玄妙電磁理，隔空傳播影與音」。1995 年擔任台揚電子工程師，隨後到景文科技大學電子系擔任講師(1995-2002)、副教授(2002-2006)、教授(2006-)，電子系主任(2005-2011)、電資學院第二任院長(2011-2017)，並曾擔任躍登科技的副總經理，他是國內首屈一指的手機天線與高頻電路設計專家，近年來則致力於 RFID 方面的研究與應用。





吳維揚博士：1999 年文大電機系學士、2001 年中山大學電機所碩士、2006 年中山大學電機所博士。2006 年進入宏達電子公司擔任工程師，從事手機天線、高頻電子電路的設計工作。2018 進入 Google 公司服務。



莊嶸騰博士：2000 年文大電機系學士、2002 年北科大機電所碩士、2006 年北科大機電所博士。專長為智慧型天線設計與車用電子設計，2006 年進入工研院資通所擔任副研究員，之後進入財團法人車輛研究測試中心綠能處擔任技術發展專案經理。

光電領域的華人大師

Father of Optical-Fiber Communication: K. Kao (高錕)



高錕博士生於上海，其後移居香港，肄業於聖約瑟書院。他在英國取得倫敦大學理學士和哲學博士學位，歷任歐美多家著名電訊機構及實驗室的重要職位，也是香港中文大學前校長。他在 1966 年發表論文，提出如何實現以玻璃纖維作為導體，令光代替電流傳遞訊息，引進光通信的時代，因此被



譽為「光纖通訊之父」，2009 年獲得諾貝爾物理獎。

Father of Integrated Optics: P. K. Tien (田炳耕)



田炳耕博士，浙江上虞人，1919 年生，美國工程院院士和美國科學院院士。他在上海中法工專畢業後，於 1937 年考入重慶國立中央大學電機工程系，1940 年返滬轉讀上海交通大學，於 1941 年畢業。1947 年田炳耕去美國史丹福(Stanford)大學完成博士課程，在 Stanford 大學發明了微波放大的新元件空間電荷波放大器 (Space charge wave Amplifier)，該放大器發表在 1952 年美國

Proceeding of IRE 的第 40 卷 688 頁上。1952 年進入美國貝爾實驗室(Bell Lab)工作，發表了多篇研究論文，並取得多項專利。這些論文和專利涉及微波理論和技術、材料科學、電波傳播、雜訊理論、鐵磁體、超導體、聲電效應，雷射物理、積體光學、高速電子學等。1975 年田炳耕當選為美國工程院院士，1978 年又當選為美國科學院院士，2017 年 12 月 27 日於美國辭世。田炳耕博士被譽為「積體光學之父」。

Godfather of A-O Devices: C. S. Tsai (蔡振水)

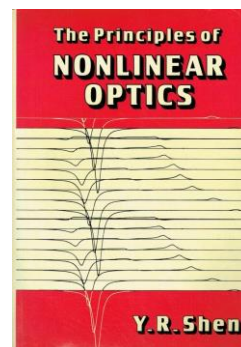


蔡振水博士，1935 年生，1957 年台大電機系學士，1965 年獲得美國史丹福大學電機工程博士。隨後在洛克希德飛機飛彈公司研究中心 (Lockheed Palo Alto Research Center) 工作三年半，1969 年應聘美國卡乃基美濃大學 (Carnegie-Mellon University) 電機工程系助理教授，1974 年晉昇正教授，並於 1979 年膺得傑出教授。蔡博士 1980 年應聘加州大學爾灣分校(Univ. of Calif., Irvine)電機工程系資深教授，1985 年擔任系主任一年半，並於 1991 年榮膺傑出教授。蔡教授之研究領域包括積體聲光學、積體磁光學、磁性微波元件、超音波顯微學等，論文及專著豐富，並獲得教研獎賞十多項。

Godfather of Nonlinear Optics: Y. R. Shen (沈元壤)



沈元壤博士生於 1935 年，台灣大學電機學士、美國史丹福大學碩士、哈佛大學博士，曾任加州大學柏克萊分校助教授、副教授，於 1970 年升任教授；1964 年起任該校勞倫斯實驗室材料化學科學組計畫主持人，1984 年起加入該校超級材料中心研究計畫擔任主持人。沈院士曾



獲查理湯斯獎(Charles Tomes Award)、非線性光學最高獎、美國能源部固態物理特殊科學成就獎、勞倫斯柏克萊傑出技術轉移獎等殊榮。1989 年其大作 The Principle of Nonlinear Optics 獲獎，並獲得美國多層光學記憶系體專利。民國 1990 年當選中研院十八屆院士。他的研究領域包括非線性光學、雷射光譜學、表面科學、凝聚體物理學等，是液晶非線性光學與表面非線性光學研究的開拓者。

Godfather of Optoelectronic/Quantum Electronics Engineering: C. L. Tang (湯仲良)

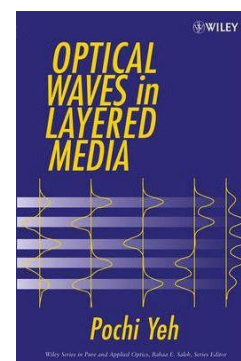


湯仲良博士生於 1934 年 5 月 14 日，1955 年獲得華盛頓大學學士，1956 年獲得加州理工學院(California Institute of Technology)碩士，1960 年獲得哈佛大學博士。1964 年後任教康乃爾大學，為世界著名的光電專家，在量子電子學方面有重大貢獻，1986 年當選美國國家工程學院院士，1994 年並當選第二十屆中研院院士。

Famous Scholar of Optoelectronics: Pochi Yeh (葉伯琦)



葉伯琦博士著有 Optical waves in layered media、Optical waves in crystals (與 Yariv 合著) 等光電經典著作，為世界級著名的光電學者。

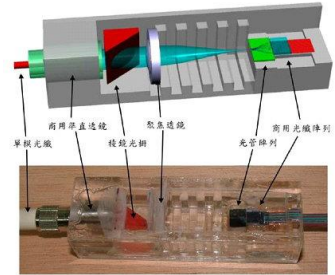


文大電機系在光電領域的系友名人



施至柔博士：1997 年文大電機系學士、1999 年交大光電碩士、2005 年交大光電博士，研究液晶顯示器、光纖通訊系統與

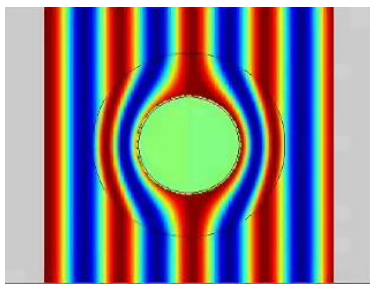
元件等。2005 年進入新竹國家實驗研究院儀器設計中心擔任研究員。



林宇仁：1999 年文大電機系學士、2001 年中央大學光電碩士。2001-2007 在新竹國家研究院儀器設計中心擔任助理研究員，之後進



入陽程科技擔任副理，後來又在玉晶光電擔任經理。期間並在台灣光學協會擔任「光學系統設計」講師，並且在台灣與大陸之間許多大學擔任光學系統設計之授課工作。而台灣第一個超過五百萬畫素的手機鏡頭，即為林宇仁所設計。



張高德博士：2001 年文大電機系學士、2003 年中央大學光電碩士、2007 年中央大學光電博士，專長為光子晶體與奈米光學。2007 後進入工研院工作，現任機械與機電系統研究所副理。



楊惠婷：1999 年文大電機系學士、2001 年台大電機碩士，台科大色彩與照明研究所博士班肄業。現任影像色彩處理工程師。

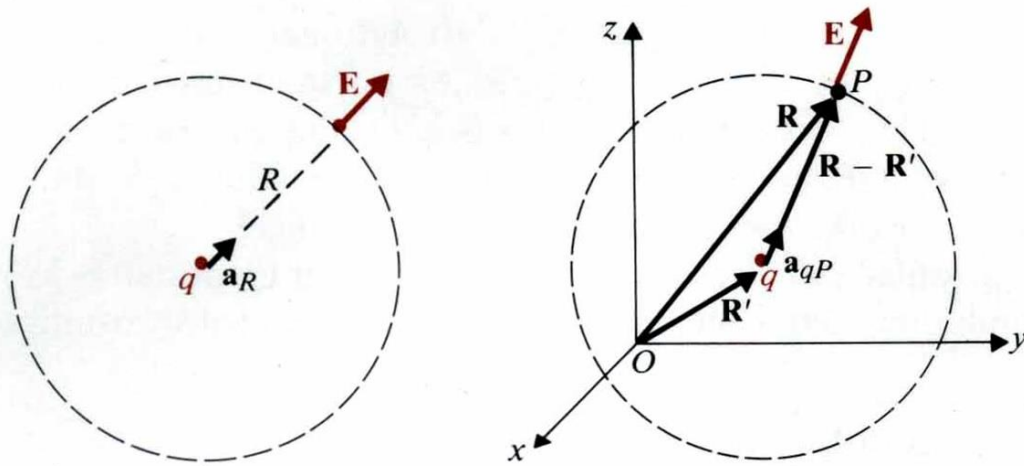


Chapter 1 Electromagnetic Field Theory

1-1 Electric Fields and Electric Dipoles

Gauss's law : $\oiint_s \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon} = \frac{1}{\epsilon} \iiint_{v'} \rho dv'$ $\xrightarrow{\text{divergence theorem}}$ $\iiint_{v'} \nabla \cdot \vec{E} dv' = \frac{1}{\epsilon} \iiint_{v'} \rho dv' \Rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$

and $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ (F/m) in the free space.



(a) Point charge at the origin.

(b) Point charge not at the origin.

For q at \vec{R}' , field point at $\vec{R} \Rightarrow \vec{E} = \frac{q \hat{a}_{RR'}}{4\pi\epsilon |\vec{R} - \vec{R}'|^2} = \frac{q(\vec{R} - \vec{R}')}{4\pi\epsilon |\vec{R} - \vec{R}'|^3}$ and $\hat{a}_{RR'} = \frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|}$.

\vec{E} due to a system of discrete charges: $\vec{E} = \frac{1}{4\pi\epsilon} \sum_{k=1}^n \frac{q_k (\vec{R} - \vec{R}'_k)}{|\vec{R} - \vec{R}'_k|^3}$

Volume source $\rho \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon} \iiint_{v'} \hat{a}_R \frac{\rho}{R^2} dv' = \frac{1}{4\pi\epsilon} \iiint_{v'} \frac{\rho \vec{R}}{|\vec{R}|^3} dv'$

Surface source $\rho_s \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon} \iint_{s'} \hat{a}_R \frac{\rho_s}{R^2} dS'$ Line source $\rho_l \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon} \int_{l'} \hat{a}_R \frac{\rho_l}{R^2} dl'$

Eg. Show that Coulomb's law $\vec{F} = \hat{a}_R \frac{q_1 q_2}{4\pi\epsilon R^2} = \frac{q_1 q_2 \vec{R}}{4\pi\epsilon R^3}$, where $\hat{a}_R = \frac{\vec{R}}{R}$, $R = |\vec{R}|$.

(Proof) $\because \vec{F} = q_2 \vec{E}$ and $\oiint \vec{E} \cdot d\vec{S} = \frac{q_1}{\epsilon} = 4\pi r^2 E$, $\vec{E} = \hat{a}_R \frac{q_1}{4\pi\epsilon R^2} = \frac{q_1 \vec{R}}{4\pi\epsilon R^3}$

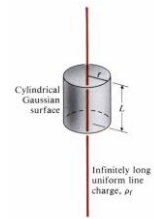
$$\therefore \vec{F} = \hat{a}_R \frac{q_1 q_2}{4\pi\epsilon R^2}$$

Eg. Determine the electric field intensity of an infinitely long line charge of a uniform density ρ_l in air.

(Sol.)

$$\oiint_S \vec{E} \cdot d\vec{S} = \int_0^L \int_0^{2\pi} E r d\phi dz = 2\pi r L E$$

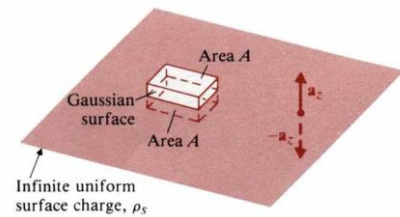
$$2\pi r L E = \frac{\rho_l L}{\epsilon_0}, \quad \vec{E} = \hat{a}_r \frac{\rho_l}{2\pi\epsilon_0 r}$$



Eg. Determine the electric field intensity of an infinite planar charge with a uniform surface charge density ρ_s .

(Sol.) $\oiint_S \vec{E} \cdot d\vec{S} = 2ES = 2EA, \quad 2EA = \frac{\rho_s A}{\epsilon_0}$

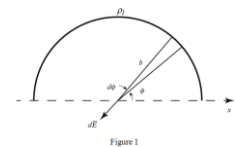
$$\Rightarrow \vec{E} = \begin{cases} = \hat{z} \frac{\rho_s}{2\epsilon_0}, & z > 0 \\ = -\hat{z} \frac{\rho_s}{2\epsilon_0}, & z < 0 \end{cases}$$



Eg. A line charge of uniform density ρ_l in free space forms a semicircle of radius b . Determine the magnitude and direction of the electric field intensity at the center of the semicircle. [高考]

(Sol.) $d\vec{E}_y = -\frac{\rho_l (bd\phi)}{4\pi\epsilon_0 b^2} \cdot \sin\phi,$

$$\vec{E} = \hat{y} E_y = -\hat{y} \frac{\rho_l}{4\pi\epsilon_0 b} \int_0^\pi \sin\phi d\phi = -\hat{y} \frac{\rho_l}{2\pi\epsilon_0 b}$$



Eg. Determine the electric field caused by spherical cloud of electrons with a volume charge density $\rho = -\rho_0$ for $0 \leq R \leq b$ (both ρ_0 and b are positive) and $\rho = 0$ for $R > b$. [交大電子物理所]

(Sol.)

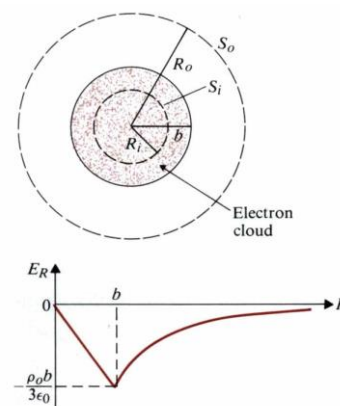
(a) $R \geq b$

$$Q = -\rho_0 \frac{4\pi}{3} b^3, \quad \vec{E} = \hat{a}_R \frac{Q}{4\pi\epsilon_0 R^2} = -\hat{a}_R \frac{\rho_0 b^3}{3\epsilon_0 R^2}$$

(b) $0 \leq R \leq b$

$$\vec{E} = \hat{a}_R E, \quad d\vec{S} = \hat{a}_R dS, \quad \oiint_{S_i} \vec{E} \cdot d\vec{S} = E \int_{S_i} dS = E 4\pi R^2$$

$$Q = \iiint_V \rho dv = -\rho_0 \iiint_V dv = -\rho_0 \frac{4\pi}{3} R^3, \quad \vec{E} = -\hat{a}_R \frac{\rho_0 R}{3\epsilon_0}$$

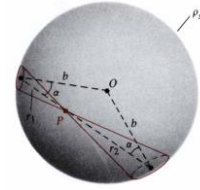


Eg. A total charge Q is put on a thin spherical shell of radius b . Determine the electrical field intensity at an arbitrary point inside the shell. [台大電研]

(Sol.)

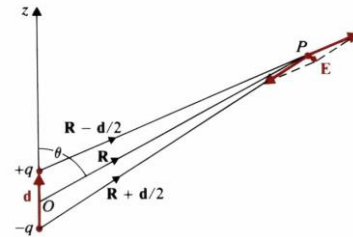
$$\rho_s = \frac{Q}{4\pi b^2}, \quad dE = \frac{\rho_s}{4\pi\epsilon_0} \left(\frac{dS_1}{r_1^2} - \frac{dS_2}{r_2^2} \right)$$

$$d\Omega = \frac{dS_1}{r_1^2} \cos\alpha = \frac{dS_2}{r_2^2} \cos\alpha, \quad dE = \frac{\rho_s}{4\pi\epsilon_0} \left(\frac{d\Omega}{\cos\alpha} - \frac{d\Omega}{\cos\alpha} \right) = 0$$



Electric dipole: A pair of equal but opposite charges with separation.

$$\begin{aligned} \left| \vec{R} - \frac{\vec{d}}{2} \right|^{-3} &= \left[\left(\vec{R} - \frac{\vec{d}}{2} \right) \cdot \left(\vec{R} - \frac{\vec{d}}{2} \right) \right]^{-3/2} = \left[R^2 - R \cdot d + \frac{d^2}{4} \right]^{-3/2} \\ &\cong R^{-3} \left[1 - \frac{\vec{R} \cdot \vec{d}}{R^2} \right]^{-3/2} \cong R^{-3} \left[1 + \frac{3}{2} \frac{\vec{R} \cdot \vec{d}}{R^2} \right] \end{aligned}$$



$$\left| \vec{R} + \frac{\vec{d}}{2} \right|^{-3} \cong R^{-3} \left[1 - \frac{3}{2} \frac{\vec{R} \cdot \vec{d}}{R^2} \right]$$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon} \left\{ \frac{\vec{R} - \frac{\vec{d}}{2}}{\left| \vec{R} - \frac{\vec{d}}{2} \right|^3} - \frac{\vec{R} + \frac{\vec{d}}{2}}{\left| \vec{R} + \frac{\vec{d}}{2} \right|^3} \right\} \cong \frac{q}{4\pi\epsilon R^3} \left[3 \frac{\vec{R} \cdot \vec{d}}{R^2} \vec{R} - \vec{d} \right] = \frac{1}{4\pi\epsilon R^3} \left[3 \frac{\vec{R} \cdot \vec{P}}{R^2} \vec{R} - \vec{P} \right]$$

$$\left(\begin{array}{l} \vec{p} = \hat{z} p = p \left(\hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta \right) \\ \vec{R} \cdot \vec{p} = p \cos\theta \end{array} \right)$$

$$\Rightarrow \vec{E} = \frac{p}{4\pi\epsilon R^3} \left(\hat{a}_r 2\cos\theta + \hat{a}_\theta \sin\theta \right)$$

Eg. At what value of θ does the electric field intensity of a z -directed dipole have no z -component.

(Sol.) $\vec{E} = \frac{p}{4\pi\epsilon_0 R^3} (\hat{a}_r 2\cos\theta + \hat{a}_\theta \sin\theta), \quad \hat{z} = \hat{a}_r \cos\theta - \hat{a}_\theta \sin\theta$

No z -component $\Rightarrow 2\cos\theta \cdot \cos\theta - \sin\theta \cdot \sin\theta = 0 \Rightarrow \tan^2 \theta = 2 \Rightarrow \theta = 54.7^\circ$ or 125.3°

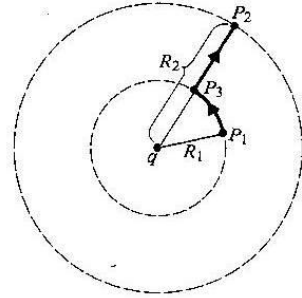
1-2 Static Electric Potentials

$$\vec{E} = -\nabla V \Rightarrow V_2 - V_1 = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} \quad \text{and} \quad \nabla^2 V = -\rho/\epsilon$$

Electric potential due to a point charge:

$$V = -\int_{\infty}^R \hat{a}_R \frac{q}{4\pi\epsilon R^2} \cdot \hat{a}_R dR = \frac{q}{4\pi\epsilon R}$$

$$V_{21} = V_{P_2} - V_{P_1} = \frac{q}{4\pi\epsilon} \left(\frac{1}{R_2} - \frac{1}{R_1} \right)$$



Electric potential due to discrete charges:

$$V = \frac{1}{4\pi\epsilon} \sum_{k=1}^n \frac{q_k}{|R - R'_k|}$$

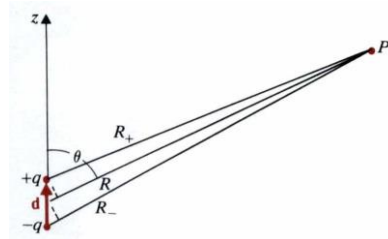
Electric potential due to an electric dipole:

$$V = \frac{q}{4\pi\epsilon} \left(\frac{1}{R_+} - \frac{1}{R_-} \right)$$

If $d \ll R$, we have

$$\frac{1}{R_+} \cong \left(R - \frac{d}{2} \cos\theta \right)^{-1} \cong R^{-1} \left(1 + \frac{d}{2R} \cos\theta \right)$$

and $\frac{1}{R_-} \cong \left(R + \frac{d}{2} \cos\theta \right)^{-1} \cong R^{-1} \left(1 - \frac{d}{2R} \cos\theta \right)$



$$V = \frac{qd \cos\theta}{4\pi\epsilon R^2}, \quad \vec{p} = q\vec{d} \Rightarrow V = \frac{\vec{p} \cdot \hat{a}_R}{4\pi\epsilon R^2} \quad (V)$$

$$\Rightarrow \vec{E} = -\nabla V = -\hat{a}_R \frac{\partial V}{\partial R} - \hat{a}_\theta \frac{\partial V}{R \partial \theta} = \frac{p}{4\pi\epsilon R^3} (\hat{a}_R 2 \cos\theta + \hat{a}_\theta \sin\theta)$$

Scalar electric potential due to various charge distributions:

Volume source $\rho \Rightarrow V = \frac{1}{4\pi\epsilon} \iiint_{v'} \frac{\rho}{R} dv'$

Surface source $\rho_s \Rightarrow V = \frac{1}{4\pi\epsilon} \iint_{s'} \frac{\rho_s}{R} dS'$

Line source $\rho_l \Rightarrow V = \frac{1}{4\pi\epsilon} \int \frac{\rho_l}{R} dl'$

Note: 1. V is a scalar, but \vec{E} is a vector.

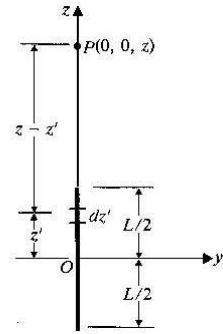
2. $\vec{E} = -\nabla V$ is valid only in the static EM field.

Eg. Obtain a formula for electrical field intensity along the axis of a uniform line charge of length L . The uniform line-charge density is ρ_l . [高考]

(Sol.) $R = z - z'$, $z > \frac{L}{2}$

$$V = \frac{\rho_l}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{dz'}{z - z'} = \frac{\rho_l}{4\pi\epsilon_0} \ln \left[\frac{z + (L/2)}{z - (L/2)} \right], \quad z > \frac{L}{2}$$

$$\vec{E} = -\hat{z} \frac{dV}{dz} = \hat{z} \frac{\rho_l L}{4\pi\epsilon_0 [z^2 - (L/2)^2]}, \quad z > \frac{L}{2}$$



Eg. A finite line charge of length L carrying uniform line charge density ρ_l is coincident with the x -axis. Determine V and \vec{E} in the plane bisecting the line charge.

(Sol.) $V = \int_{-L/2}^{L/2} \frac{\rho_l dx}{4\pi\epsilon_0 \sqrt{x^2 + y^2}} = \frac{\rho_l}{2\pi\epsilon_0} \left\{ \ln \left[\sqrt{\left(\frac{L}{2}\right)^2 + y^2} + \frac{L}{2} \right] - \ln y \right\}$

and $\vec{E} = -\nabla V = \hat{y} \frac{\rho_l}{2\pi\epsilon_0 y} \left[\frac{L/2}{\sqrt{(L/2)^2 + y^2}} \right]$

Eg. A charge is distributed uniformly over an $L \times L$ square plate. Determine V and \vec{E} at a point on the axis perpendicular to the plate and through its center.

(Sol.) $\rho_s = \frac{Q}{L^2}$, $y^2 \rightarrow y^2 + z^2$, $V = \frac{\rho_s}{2\pi\epsilon_0} \int_{-L/2}^{L/2} \left\{ \ln \left[\sqrt{\left(\frac{L}{2}\right)^2 + y^2 + z^2} + \frac{L}{2} \right] - \ln \sqrt{y^2 + z^2} \right\} dy$

$$= \frac{Q}{\pi\epsilon_0 L^2} \left\{ \frac{L}{2} \ln \left[\frac{\sqrt{2\left(\frac{L}{2}\right)^2 + z^2} + \frac{L}{2}}{\sqrt{2\left(\frac{L}{2}\right)^2 + z^2} - \frac{L}{2}} \right] - z \cdot \tan^{-1} \left[\frac{\left(\frac{L}{2}\right)^2}{z \sqrt{2\left(\frac{L}{2}\right)^2 + z^2}} \right] \right\}$$

$$\vec{E} = -\nabla V = \hat{z} \frac{Q}{\pi\epsilon_0 L^2} \tan^{-1} \left[\frac{\left(\frac{L}{2}\right)^2}{z \sqrt{2\left(\frac{L}{2}\right)^2 + z^2}} \right]$$

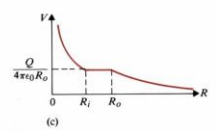
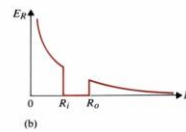
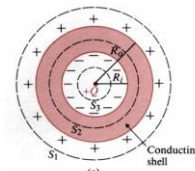
Eg. A positive point charge Q is at the center of a spherical conducting shell of an inner radius R_i and an outer radius R_o . Determine \vec{E} and V as functions of the radial distance R . [高考]

(Sol.) $R > R_o$, $\oiint_s \vec{E} \cdot d\vec{S} = E4\pi R^2 = \frac{Q}{\epsilon_0}$, $E = \frac{Q}{4\pi\epsilon_0 R^2}$, $V = -\int_{\infty}^R E dR = \frac{Q}{4\pi\epsilon_0 R}$

$R_i < R < R_o$, $E = 0$, $V = V|_{R=R_o} = \frac{Q}{4\pi\epsilon_0 R_o}$

$R < R_i$, $E = \frac{Q}{4\pi\epsilon_0 R^2}$, $V = -\int E dR + C = \frac{Q}{4\pi\epsilon_0 R} + C$

$C = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_o} - \frac{1}{R_i} \right) \Rightarrow V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R} + \frac{1}{R_o} - \frac{1}{R_i} \right)$

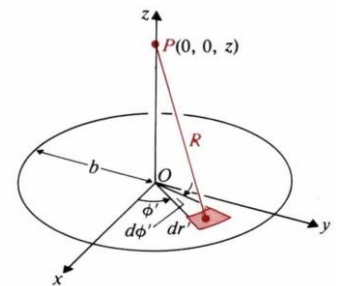


Eg. Obtain a formula for the electric field intensity on the axis of a circular disk of radius b that carries uniform surface charge density ρ_s . [高考]

(Sol.) $ds' = r' dr' d\phi'$, $R = \sqrt{z^2 + r'^2}$

$V = \frac{\rho_s}{4\pi\epsilon_0} \int_0^{2\pi} \int_0^b \frac{r'}{(z^2 + r'^2)^{3/2}} dr' d\phi' = \frac{\rho_s}{2\epsilon_0} \left[(z^2 + b^2)^{1/2} - |z| \right]$

$\vec{E} = -\nabla V = -\hat{z} \frac{\partial V}{\partial z} = \begin{cases} \hat{z} \frac{\rho_s}{2\epsilon_0} \left[1 - z(z^2 + b^2)^{-1/2} \right] & z > 0 \\ -\hat{z} \frac{\rho_s}{2\epsilon_0} \left[1 + z(z^2 + b^2)^{-1/2} \right] & z < 0 \end{cases}$



Eg. Make a two-dimensional sketch of the equipotential lines and the electric field lines for an electric dipole.

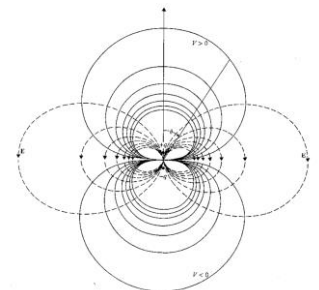
(Sol.)

For an electric dipole, $V = \frac{qd \cos\theta}{4\pi\epsilon_0 R^2} = \text{constant} \Rightarrow R = c_v \sqrt{\cos\theta}$

$d\vec{l} = k\vec{E}$, where k is a constant.

$\hat{a}_R dR + \hat{a}_\theta R d\theta + \hat{a}_\phi R \sin\theta d\phi = k \left(\hat{a}_R E_R + \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi \right)$

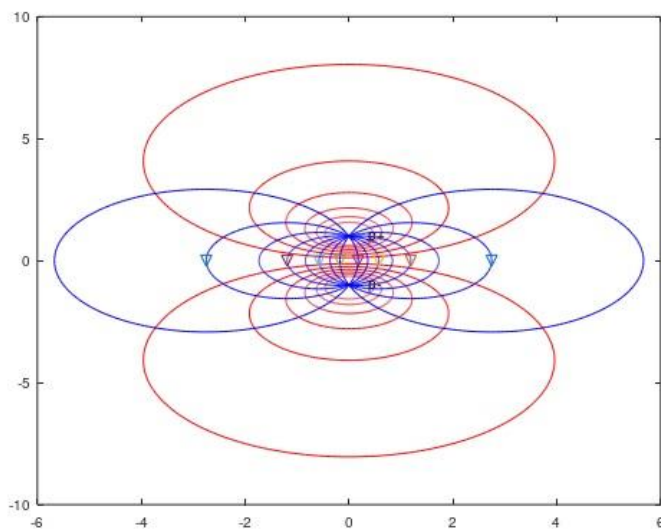
$\frac{dR}{E_R} = \frac{R d\theta}{E_\theta} = \frac{R \sin\theta d\phi}{E_\phi}$, $\frac{dR}{2 \cos\theta} = \frac{R d\theta}{\sin\theta}$, $R = c_E \sin^2 \theta$



```

u=0.5:0.5:3;
c=exp(-u/2);
xc=(c.^2+1)/(c.^2-1);
r=2*c./(c.^2-1);
phi=linspace(0,2*pi,200);
X=cos(phi)*r;
Y=sin(phi)*r;
XC=ones(size(phi))*xc;
plot(X,Y+XC,'r');
text(0,1,'\leftarrow p+');
hold on
plot(X,Y-XC,'r');
text(0,-1,'\leftarrow p-');
th=(20:20:160)*pi/180;
c=cot(th);
r=sqrt(1+c.*c);
X=cos(phi)*r;
Y=sin(phi)*r;
C=ones(size(phi))*c;
plot(X+C,Y,'b');
plot(c,0,'v');
xline(0,'|')
yline(0,'-')
title('equipotential lines(red line) and the electric field lines(blue line) for an electric
dipole');

```



1-3 Magnetic Fields

Magnetic field: $\vec{H} = \frac{\vec{B}}{\mu} = \frac{\vec{B}}{\mu_0} - \vec{M}$ (A/m), where $\mu_0 = 4\pi \times 10^{-7}$ in the free space.

Magnetic flux density: $\vec{B} = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\vec{\ell} \times \hat{a}_R}{R^2}$

Ampere's law of \vec{H} : $\oint \vec{H} \cdot d\vec{l} = I \Leftrightarrow \nabla \times \vec{H} = \vec{J}$

$$\frac{1}{\mu_0} \nabla \times \vec{B} = \vec{J} + \vec{J}_m = \vec{J} + \nabla \times \vec{M} \quad \text{or} \quad \nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J},$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow \oint \vec{H} \cdot d\vec{l} = I, \quad \text{and} \quad \vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_m) \vec{H} = \mu_0 \mu_r \vec{H},$$

where $\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0}$

Gauss's law of \vec{B} : $\nabla \cdot \vec{B} = 0 \Leftrightarrow \oiint_S \vec{B} \cdot d\vec{S} = 0$

$\therefore \nabla \cdot \vec{B} = 0, \exists \vec{A}$ fulfills $\vec{B} = \nabla \times \vec{A}$

$$\nabla \times \vec{B} = \mu \vec{J} = \nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

Choose $\nabla \cdot \vec{A} = 0 \Rightarrow \nabla^2 \vec{A} = -\mu \vec{J}$ (Note: $\nabla^2 V = -\frac{\rho}{\epsilon}$ is scalar Poisson's equation)

$$\therefore V = \frac{1}{4\pi\epsilon} \iiint_{V'} \frac{\rho}{R} dv', \quad \therefore \vec{A} = \frac{\mu}{4\pi} \iiint_{V'} \frac{\vec{J}}{R} dv' \quad (\text{Wb/m})$$

Magnetic flux: $\Phi = \iint_S \vec{B} \cdot d\vec{S} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{S} = \oint_C \vec{A} \cdot d\vec{l}' \quad (\text{Wb})$

Biot-Savart's law: $\vec{B} = \frac{\mu I}{4\pi} \oint \frac{d\vec{\ell}' \times \hat{a}_R}{R^2} = \frac{\mu I}{4\pi} \oint_{C'} \frac{d\vec{\ell}' \times \vec{R}}{R^3}$

$$\vec{A} = \frac{\mu}{4\pi} \iiint_{V'} \frac{\vec{J}}{R} dv' = \frac{\mu I}{4\pi} \oint_{C'} \frac{d\vec{\ell}'}{R}, \quad \therefore \nabla \times (f\vec{G}) = f\nabla \times \vec{G} + (\nabla f) \times \vec{G},$$

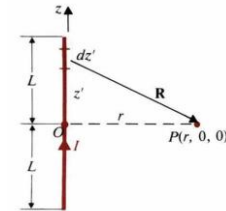
$$\therefore \vec{B} = \nabla \times \vec{A} = \nabla \times \left[\frac{\mu I}{4\pi} \oint_{C'} \frac{d\vec{\ell}'}{R} \right] = \frac{\mu I}{4\pi} \oint_{C'} \nabla \times \left(\frac{d\vec{\ell}'}{R} \right) = \frac{\mu I}{4\pi} \oint_{C'} \left[\frac{1}{R} \nabla \times d\vec{\ell}' + \left(\nabla \frac{1}{R} \right) \times d\vec{\ell}' \right] = \frac{\mu I}{4\pi} \oint_{C'} \frac{d\vec{\ell}' \times \hat{a}_R}{R^2}$$

Note: $\begin{cases} \nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \\ d\vec{l}' = \hat{x}dx' + \hat{y}dy' + \hat{z}dz' \Rightarrow \nabla \times d\vec{l}' = 0 \end{cases}$, and then $d\vec{B} = \frac{\mu I}{4\pi} \left(\frac{d\vec{l}' \times \hat{a}_R}{R^2} \right) = \frac{\mu I}{4\pi} \frac{d\vec{l}' \times \vec{R}}{R^3}$

Eg. A direct current I flows in a straight wire of length $2L$. Find the magnetic flux density \vec{B} at a point located at a distance r from the wire in the bisecting plane.

(Sol.) $d\vec{l}' \times \vec{R} = \hat{z} dz' \times (\hat{a}_r r - \hat{z} z') = \hat{a}_\phi r dz'$, $R = (z^2 + r^2)^{1/2}$

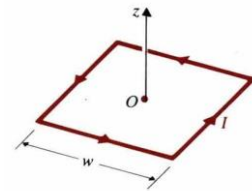
$$\vec{B} = \hat{a}_\phi \frac{\mu_0 I}{4\pi} \int_{-L}^L \frac{r dz'}{(z'^2 + r^2)^{3/2}} = \hat{a}_\phi \frac{\mu_0 I L}{2\pi r \sqrt{L^2 + r^2}}$$



Eg. Find the magnetic flux density at the center of a square loop, with side w carrying a direct current I .

(Sol.) $L = \frac{w}{2}$, $r = \frac{w}{2}$ in this case,

$$\vec{B} = 4 \times \hat{z} \frac{\mu_0 I \frac{w}{2}}{2\pi \frac{w}{2} \sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{w}{2}\right)^2}} = \hat{z} \frac{2\sqrt{2}\mu_0 I}{\pi w}$$

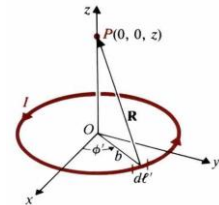


Eg. Find the magnetic flux density at a point on the axis of a circular loop of radius b that a direct current I .

(Sol.) $d\vec{l}' = \hat{a}_\phi b d\phi'$, $\vec{R} = \hat{z}z - \hat{a}_r b$, $R = (z^2 + b^2)^{1/2}$

$$d\vec{l}' \times \vec{R} = \hat{a}_r b z d\phi' + \hat{z} b^2 d\phi'$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{\hat{a}_r b z + \hat{z} b^2}{(z^2 + b^2)^{3/2}} d\phi' = \hat{z} \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}}. \text{ In case of } z=0, \vec{B} = \hat{z} \frac{\mu_0 I}{2b}$$



Eg. Determine the magnetic flux density at a point on the axis of a solenoid with radius b and length L , and with a current in its N turns of closely wound coil.

(Sol.) $d\vec{B} = \frac{\hat{z} \mu_0 I b^2}{2[(z - z')^2 + b^2]^{3/2}} \left(\frac{N}{L} dz' \right)$

$$\vec{B} = \int_0^L d\vec{B} = \frac{\mu_0 N I}{2L} \left[\frac{L - z}{[(L - z)^2 + b^2]^{1/2}} + \frac{z}{\sqrt{z^2 + b^2}} \right] = \frac{\mu_0 N I}{2L} \left[\frac{z}{\sqrt{z^2 + b^2}} - \frac{z - L}{\sqrt{(z - L)^2 + b^2}} \right]$$

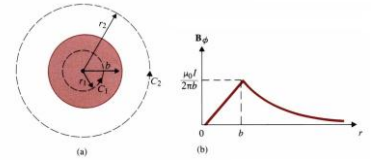
Ampere's law of \vec{B} : $\nabla \times \vec{B} = \mu \vec{J} \Leftrightarrow \oint_c \vec{B} \cdot d\vec{l} = \mu I$, where $\mu = \mu_0$ in the free space.

Eg. An infinitely long, straight conductor with a circular cross section of radius b carries a steady current I . Determine the magnetic flux density both inside and outside the conductor. [交大光電所]

(Sol.)

(a) Inside the conductor, $r \leq b$:

$$\oint_{C_1} \vec{B} \cdot d\vec{l} = \int_0^{2\pi} B r d\phi = 2\pi r B = \mu_0 \left(\frac{\pi r^2}{\pi b^2}\right) I = \mu_0 \left(\frac{r}{b}\right)^2 I \Rightarrow \vec{B} = \hat{a}_\phi \frac{\mu_0 I}{2\pi b^2} r$$

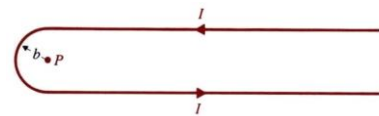


(b) Outside the conductor: $\oint_{C_2} \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I \Rightarrow \vec{B} = \hat{a}_\phi \frac{\mu_0 I}{2\pi r}$

Eg. A long line carrying a current I folds back with semicircular bend of radius b . Determine magnetic flux density at the center point P of the bend. [高考]

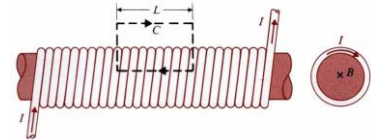
(Sol.)

$$\vec{B} = \vec{B}_1 + \vec{B}_2, \text{ where } \vec{B}_1 = 2 \cdot \hat{z} \frac{\mu_0 I}{4\pi b}, \vec{B}_2 = \hat{z} \frac{\mu_0 I}{4b}$$



Eg. Determine the magnetic flux density inside an infinitely long solenoid with air core having n closely wound turns per unit length and carrying a current I .

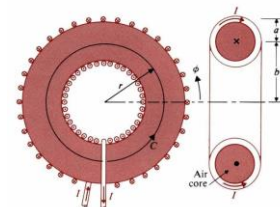
(Sol.) $BL = \mu_0 n L I \Rightarrow B = \mu_0 n I$



Eg. Determine the magnetic flux density inside a closely wound toroidal coil with an air core having N turns and carrying a current I . The toroid has a mean radius b , and the radius of each turn is a .

(Sol.) $\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 N I$

$$\vec{B} = \begin{cases} \hat{a}_\phi B = \hat{a}_\phi \frac{\mu_0 N I}{2\pi r}, & b-a < r < b+a \\ 0, & \text{elsewhere} \end{cases}$$



Eg. In certain experiments it is desirable to have a region of constant magnetic flux density. This can be created in an off-center cylindrical cavity. The uniform axial current density is $\vec{J} = \hat{z}J$. Find the magnitude and direction of \vec{B} in the cylindrical cavity whose axis is displaced from that of the conducting part by a distance d . [台大電研、清大電研、中原電機]

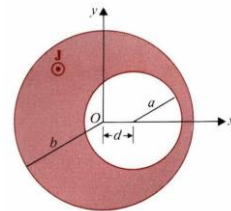
(Sol.) $\vec{J} = \hat{z}J$, $\oint \vec{B} \cdot d\vec{l} = \mu_o I$

If no hole exists, $2\pi r_1 B_{\phi 1} = \mu_o \pi r_1^2 J \Rightarrow B_{\phi 1} = \frac{\mu_o r_1}{2} J \Rightarrow \begin{cases} B_{x1} = -\frac{\mu_o J}{2} y_1 \\ B_{y1} = \frac{\mu_o J}{2} x_1 \end{cases}$

For $-\vec{J}$ in the hole portion, $B_{\phi 2} = -\frac{\mu_o r_2}{2} J \Rightarrow \begin{cases} B_{x2} = \frac{\mu_o J}{2} y_2 \\ B_{y2} = -\frac{\mu_o J}{2} x_2 \end{cases}$

At $y_1 = y_2$ and $x_1 = x_2 + d \Rightarrow B_x = B_{x1} + B_{x2} = 0$, and

$$B_y = B_{y1} + B_{y2} = \frac{\mu_o J}{2} d$$



1-4 Electromagnetic Forces

Lorentz force equation: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Electric force: $\vec{F}_e = q\vec{E}$. **Magnetic force:** $\vec{F}_m = q\vec{v} \times \vec{B}$

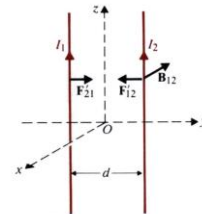
Magnetic force due to \vec{B} and I :

$$F_m = q\vec{v} \times \vec{B} \Rightarrow dF_m = dq \frac{d\vec{\ell}}{dt} \times \vec{B} = \frac{dq}{dt} d\vec{\ell} \times \vec{B} = Id\vec{\ell} \times \vec{B}, \therefore F_m = I \oint_C d\vec{\ell} \times \vec{B}$$

Eg. Determine the force per unit length between two infinitely long parallel conducting wires carrying currents I_1 and I_2 in the same direction. The wires are separated by a distance d . [清大電研]

(Sol.) $\vec{F}_{12}' = I_2(\hat{z} \times \vec{B}_{12})$,

$$\vec{B}_{12} = -\hat{x} \frac{\mu_0 I_1}{2\pi d} \Rightarrow \vec{F}_{12}' = -\hat{y} \frac{\mu_0 I_1 I_2}{2\pi d}$$



Eg. The bar AA', serves as a conducting path for the current I in two very long parallel lines. The lines have a radius b and are spaced at a distance d apart. Find the direction and the magnitude of the magnetic force on the bar. [中山物理所]

(Sol.)

$$\vec{B} = -\hat{z} \frac{\mu_0 I}{4\pi} \left(\frac{1}{y} + \frac{1}{d-y} \right), \quad d\vec{\ell} = \hat{y} dy$$

$$\Rightarrow d\vec{F} = Id\vec{\ell} \times \vec{B} = -\hat{x} \frac{\mu_0 I^2}{4\pi} \left(\frac{1}{y} + \frac{1}{d-y} \right) dy \Rightarrow \vec{F} = \int_b^{d-b} d\vec{F} = -\hat{x} \frac{\mu_0 I^2}{2\pi} \ln\left(\frac{d}{b} - 1\right)$$



Application of the electric forces: The e-paper





Scalar electric potential function: $V = \frac{1}{4\pi\epsilon} \iiint_{V'} \frac{\rho}{R} dv'$

Vector magnetic potential function: $\vec{A} = \frac{\mu}{4\pi} \iiint_{V'} \frac{\vec{J}}{R} dv'$

Retarded potentials:

$V(R,t) = \frac{1}{4\pi\epsilon} \iiint_{V'} \frac{\rho(t-R/v)}{R} dv'$, $\vec{A}(R,t) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\vec{J}(t-R/v)}{R} dv'$

1-5 Faraday's Law and Magnetic Dipoles

Faraday's law: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or $\oint_C \vec{E} \cdot d\vec{\ell} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$

$$\therefore \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \vec{A}) \Rightarrow \nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0$$

$$\therefore \exists V \text{ fulfills } \vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla V \Rightarrow \vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

Note: In static field: $\vec{E} = -\nabla V$, but in time-varying field: $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$

Emf: $V = \oint_C \vec{E} \cdot d\vec{\ell}$. **Magnetic flux:** $\Phi = \iint_S \vec{B} \cdot d\vec{S}$, $\oint_C \vec{E} \cdot d\vec{\ell} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \Rightarrow V = -\frac{d\Phi}{dt}$

Motional emf: $V' = \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$ (Volt)

$$\vec{F}_m = q\vec{v} \times \vec{B} \Rightarrow \frac{\vec{F}_m}{q} = \vec{v} \times \vec{B} \equiv -\vec{E}_m \Rightarrow V' = -\oint_C \vec{E}_m \cdot d\vec{\ell} = \oint_C (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

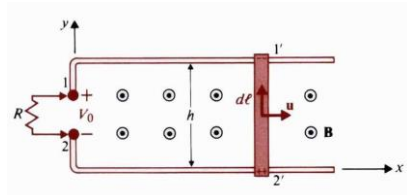
Eg. A metal bar slides over a pair of conducting rails in a uniform magnetic field

$\vec{B} = \hat{z}B_0$ with a constant velocity v . (a) Determine the open-circuit voltage V_0 that

appears across terminals 1 and 2. (b) Assuming that a resistance R is connected between the two terminals, find the electric power dissipated in R . [交大電物所]

(Sol). (a) $V_0 = V_1 - V_2 = \int_2^1 (\hat{x}v \times \hat{z}B_0) \cdot (\hat{y}d\ell) = -vB_0h$

(b) $P_e = I^2 R = \left(\frac{vB_0h}{R}\right)^2 R = \frac{(vB_0h)^2}{R}$ (W)



Eg. The circuit in Fig. is situated in a magnetic

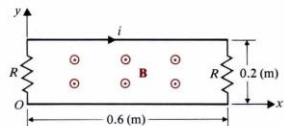
field $\vec{B} = \hat{z}3\cos(5\pi \times 10^7 t - \frac{2}{3}\pi x) \mu T$. Assuming $R=15\Omega$, find the current i . [中山物

理所]

(Sol.) $\Phi = \int_0^{0.6} 3\cos(5\pi \times 10^7 t - \frac{2}{3}\pi x)10^{-6} \cdot (0.2dx)$

$$V = -\frac{d\Phi}{dt} = 45[\cos(5\pi \times 10^7 t - \frac{2}{3}\pi \cdot 0.6) - \cos(5\pi \times 10^7 t)]$$

$$i = \frac{V}{2R} = 1.76\sin(5\pi \times 10^7 t - 0.2\pi)$$

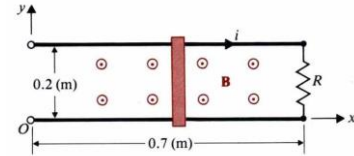


Ex. A conducting sliding bar oscillates over two parallel conducting rails in a sinusoidally varying magnetic field $\vec{B} = \hat{z}5 \cos(\omega t) \text{ T}$. The position of the sliding bar is given by $x=0.35(1-\cos\omega t)$, and the rails are terminated in a resistance $R=0.2 \text{ } \Omega$. Find i .

(Sol.) $\Phi = 5 \cos \omega t \cdot 0.2(0.7 - x)$, $x=0.35(1-\cos\omega t)$,

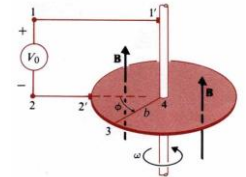
$$i = -\frac{1}{R} \frac{d\Phi}{dt}$$

$$\Rightarrow i = 1.75\omega \sin \omega t \cdot (1 + 2 \cos \omega t)$$



Ex. The Faraday disk generator consists of a circular metal disk rotating with a constant angular velocity ω in a uniform and constant magnetic field of flux density $B = \hat{z}B_0$ that is parallel to the axis of rotation. Brush contacts are the open-circuit voltage of the generator if the radius of the disk is b .

$$(\text{Sol.}) \quad V_0 = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = \int_3^4 [(\hat{a}_\phi r \omega) \times \hat{z}B_0] \cdot (\hat{a}_r dr) = \omega B_0 \int_b^0 r dr = \frac{\omega B_0 b^2}{2} \quad (\text{V})$$

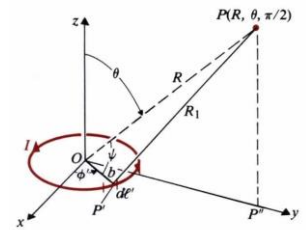


Magnetic dipole moment:

$\vec{m} = \hat{z}IS = \hat{z}m$, where S is the area of the loop that carries I and $m=IS$.

Vector potential of a magnetic dipole:

$$\vec{A} = \frac{\mu_0 \vec{m} \times \hat{a}_R}{4\pi R^2}, \text{ where } \vec{B} = \nabla \times \vec{A} = \frac{\mu_0 m}{4\pi R^3} (\hat{a}_R 2 \cos \theta + \hat{a}_\theta \sin \theta)$$



Magnetization vector: $\vec{M} = \lim_{\Delta V \rightarrow 0} \frac{\sum_{k=1}^{n\Delta V} \vec{m}_k}{\Delta V}$ (A/m), where \vec{m}_k is the

magnetic dipole moment of an atom.

$$d\vec{A} = \frac{\mu_0 \vec{M} \times \hat{a}_R}{4\pi R^2} dv' = \frac{\mu_0}{4\pi} \vec{M} \times \nabla' \left(\frac{1}{R} \right) dv' = \frac{\mu_0}{4\pi} \left[\frac{1}{R} \nabla' \times \vec{M} - \nabla' \times \left(\frac{\vec{M}}{R} \right) \right]$$

$$\Rightarrow \vec{A} = \frac{\mu_0}{4\pi} \iiint_{V'} \frac{\nabla' \times \vec{M}}{R} dv' - \frac{\mu_0}{4\pi} \iiint_{V'} \nabla' \times \left(\frac{\vec{M}}{R} \right) dv' \quad \left(\iiint_{V'} \nabla' \times \vec{F} dv' = -\iint_S \vec{F} \times d\vec{S}' \right)$$

$$= \frac{\mu_0}{4\pi} \iiint_{V'} \frac{\nabla' \times \vec{M}}{R} dv' + \frac{\mu_0}{4\pi} \iint_S \frac{\vec{M} \times \hat{a}_n'}{R} dS'$$

\therefore Magnetization volume current density: $\boxed{\vec{J}_m = \nabla \times \vec{M}}$ (A/m²)

Magnetization surface current density: $\boxed{\vec{J}_{ms} = \vec{M} \times \hat{a}_n}$ (A/m)

Equivalent Magnetization Charge Densities:

$$V_m = \frac{1}{4\pi} \oint_{S'} \frac{\vec{M} \cdot \hat{a}_n'}{R} dS' + \frac{1}{4\pi} \iiint_{V'} -\frac{(\nabla' \cdot \vec{M})}{R} dv' \quad (\text{Note: } V = \frac{1}{4\pi\epsilon_o} \left\{ \oint_{S'} \frac{\vec{P} \cdot \hat{a}_n'}{R} dS' + \frac{1}{4\pi\epsilon_o} \iiint_{V'} -\frac{(\nabla' \cdot \vec{P})}{R} dv' \right\})$$

Define the magnetization surface charge density as $\rho_{ms} = \vec{M} \cdot \hat{a}_n$ and the

magnetization volume charge density as $\rho_m = -\nabla \cdot \vec{M}$

Eg. A ferromagnetic sphere of radius b is uniformly magnetized with a magnetization $\vec{M} = \hat{z}M_0$. (a) Determine the equivalent magnetization current densities \vec{J}_m and \vec{J}_{ms} . (b) Determine the magnetic flux density at the center of the sphere. [台大電研]

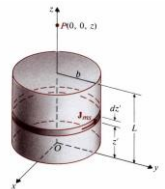
$$(\text{Sol.}) \text{ (a) } \vec{J}_m = \nabla' \times \vec{M} = 0, \quad \vec{J}_{ms} = (\hat{a}_R \cos\theta - \hat{a}_\theta \sin\theta)M_0 \times \hat{a}_R = \hat{a}_\phi M_0 \sin\theta$$

$$\text{(b) } d\vec{B} = \hat{z} \frac{\mu_o (J_{ms} b d\theta)(b \sin\theta)^2}{2(b^2)^{3/2}} = \hat{z} \frac{\mu_o M_0}{2} \sin^3 \theta d\theta, \quad \vec{B} = \hat{z} \frac{\mu_o M_0}{2} \int_0^\pi \sin^3 \theta d\theta = \hat{z} \frac{2}{3} \mu_o M_0.$$

Eg. Determine the magnetic flux density on the axis of a uniformly magnetized circular cylinder of a magnetic material. The cylinder has a radius b , length L and axial magnetization $\vec{M} = \hat{z}M_0$. [台大物研]

$$(\text{Sol.}) \quad \vec{J}_m = \nabla' \times \vec{M} = 0, \quad \vec{J}_{ms} = \vec{M} \times \hat{a}_n = (\hat{z}M_0) \times \hat{a}_r = \hat{a}_\phi M_0, \quad dI = J_{ms} dz'$$

$$\vec{B} = \hat{z} \int_0^L \frac{\mu_o M_0 b^2 dz'}{2[(z-z')^2 + b^2]^{3/2}} = \hat{z} \frac{\mu_o M_0}{2} \left[\frac{z}{\sqrt{z^2 + b^2}} - \frac{z-L}{\sqrt{(z-L)^2 + b^2}} \right]$$



Consider an infinitely long solenoid with n turns per unit length around to create a magnetic field; a voltage $V_1 = -n d\Phi/dt$ is induced unit length, which opposes the current change. Power $P_1 = -V_1 I$ per unit length must be supplied to overcome this induced voltage in order to increase the current to I . The work per unit volume required to produce a final magnetic flux density B_f is $W_1 = \int_0^{B_f} H dB$.