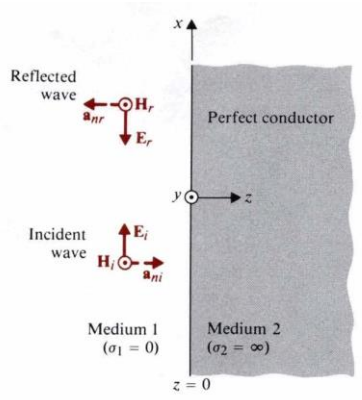
Chapter 3 Plane EM Waves and Lasers in Distinct Media

3-1 Normal Incidence at a Plane Conducting Boundary



$\hat{a}_{ni} = +\hat{z}$, $\hat{a}_{nr} = -\hat{z}$

Incident plane wave: $\vec{E}_i(z) = \hat{x}E_{i0}e^{-j\beta_1 z}$, $\vec{H}_i(z) = \hat{y}\dfrac{E_{i0}}{\eta_1}e^{-j\beta_1 z}$,

Reflective wave: $\vec{E}_r(z) = \hat{x}E_{r0}e^{j\beta_1 z}$,

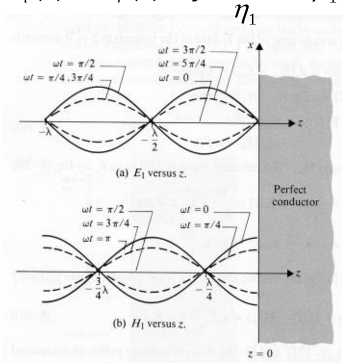
Total field: $\vec{E}_1(z) = \vec{E}_i(z) + \vec{E}_r(z)$

$\because$ Conducting boundary, $\therefore$ $\vec{E}_1(z=0) = 0 \Rightarrow E_{r0} = -E_{i0} \Rightarrow \vec{E}_r(z) = -\hat{x}\vec{E}_{i0}e^{+j\beta_1 z}$

$$\vec{H}_r(z) = \frac{1}{\eta_1}\hat{a}_{nr}\times\vec{E}_r(z) = \hat{y}\frac{E_{i0}}{\eta_1}e^{+j\beta_1 z} \Rightarrow \begin{cases}\vec{E}_1(z) = \vec{E}_i(z) + \vec{E}_r(z) = -\hat{x}j2E_{10}\sin\beta_1 z \\ \vec{H}_1(z) = \vec{H}_i(z) + \vec{H}_r(z) = \hat{y}2\dfrac{E_{i0}}{\eta_1}\cos\beta_1 z\end{cases}$$

Standing wave:

- Zero (null) of E_1 at $\beta_1 z=-n\pi$
- Maximum of H_1 at $z=-n\lambda/2$
- Zero (null) of H_1 at $\beta_1 z=-(2n+1)\pi/2$
- Maximum of E_1 at $z=-(2n+1)\lambda/4$



(a) E_1 versus z.

(b) H_1 versus z.

Eg. A y-polarized plane wave, f=100MHz, amplitude=6mV/m, propagates +x direction and impinges normally on a conducting plane at x=0. (a) Write $\vec{E}_i$, $\vec{H}_i$, $\vec{E}_r$, $\vec{H}_r$, $\vec{E}_1$, and $\vec{H}_1$. (b) Determine the location nearest to the conducting plane when $\vec{E}_1$=0.

(Sol.) $\omega = 2\pi f = 2\pi\times 10^8$, $\beta_1 = \dfrac{\omega}{c} = \dfrac{2\pi}{3}$, $\eta_1 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}} = 120\pi\Omega$, $\hat{a}_{ni} = \hat{x}$, $\hat{a}_{nr} = -\hat{x}$

(a) $\vec{E}_i(x) = \hat{y}6\times 10^{-3}e^{-j\frac{2\pi}{3}x}$, $\vec{H}_i(x) = \dfrac{1}{\eta_1}\hat{a}_{ni}\times\vec{E}_i(x) = \hat{z}\dfrac{10^{-4}}{2\pi}e^{-j\frac{2\pi}{3}x}$, $\vec{E}_r(x) = -\hat{y}6\times 10^{-3}e^{+j\frac{2\pi}{3}x}$,

$\vec{H}_r(x) = \hat{z}\dfrac{10^{-4}}{2\pi}e^{+j\frac{2\pi}{3}x}$, $\vec{E}_1(x) = \vec{E}_i(x) + \vec{E}_r(x) = -\hat{y}j12\times 10^{-3}\sin(\dfrac{2\pi}{3}x)$,

$\vec{H}_1(x) = \vec{H}_i(x) + \vec{H}_r(x) = \hat{z}\dfrac{10^{-4}}{\pi}\cos(\dfrac{2\pi}{3}x)$ (b) $\dfrac{2\pi x}{3} = -\pi$, $x = -\dfrac{3}{2}$

Eg. A right-hand circularly plane wave represented by $E(z)=E_0(\hat{x}-j\hat{y})e^{-j\beta z}$ **impinges normally on a perfectly conducting wall at** z**=0. (a) Determine the polarization of the reflected wave. (b) Find the induced current on the conducting wall.**

(Sol.) (a) $\vec{E}_r(z)=(\hat{x}E_{rx}+\hat{y}E_{ry})e^{j\beta z}$, $\vec{E}_i(0)+\vec{E}_r(0)=0\Rightarrow E_{rx}=-E_0,\ E_{ry}=jE_0$,

$\therefore\ \vec{E}_r(z)=(-\hat{x}E_0+\hat{y}jE_0)e^{j\beta z}$ is left-hand circularly-polarized and –z-propagated.

(b) $\vec{J}=\hat{a}_n\times(\vec{H}_1-\vec{H}_2)=\hat{a}_{ni}\times\vec{H}_1\ (\because \vec{H}_2=0),\ \ \hat{a}_{ni}=\hat{z},\ \ \hat{a}_{nr}=-\hat{z}$

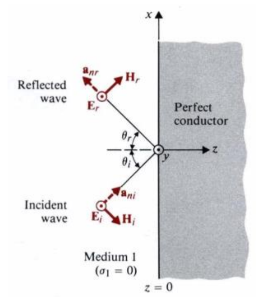
$\vec{H}_i(z)=\frac{1}{\eta_0}\hat{a}_{ni}\times\vec{E}_i=\frac{E_0}{\eta_0}(\hat{x}j+\hat{y})e^{-j\beta z}$, $\vec{H}_r(z)=\frac{1}{\eta_0}\hat{a}_{nr}\times\vec{E}_r=\frac{E_0}{\eta_0}(\hat{x}j+\hat{y})e^{j\beta z}\Rightarrow\vec{H}_1(z)=\frac{E_0}{\eta_0}(\hat{x}j+\hat{y})(e^{-j\beta z}+e^{j\beta z})$

$$\vec{J}(z)=\frac{E_0}{\eta_0}(-\hat{x}+\hat{y}j)(e^{-j\beta z}+e^{j\beta z})$$

3-2 Oblique Incidences at a Plane Conducting Boundary

Case 1 Perpendicular Polarization **(TE):**

$\hat{a}_{ni}=\hat{x}\sin\theta_i+\hat{z}\cos\theta_i$, $\hat{a}_{nr}=\hat{x}\sin\theta_r-\hat{z}\cos\theta_r$



Incidence plane wave: $\vec{E}_i(x,z)=\hat{y}E_{i0}e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)}$

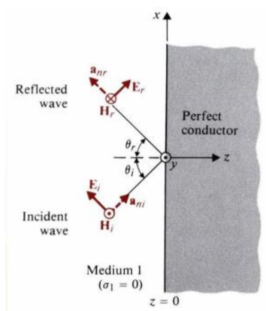
$$\vec{H}_i(x,z)=\frac{1}{\eta_1}[\hat{a}_{ni}\times\vec{E}_i(x,z)]=\frac{E_{i0}}{\eta_1}(-\hat{x}\cos\theta_i+\hat{z}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)}$$

$\because\ \vec{E}_1(x,0)=\vec{E}_i(x,0)+\vec{E}_r(x,0)=0$, $\therefore$ $E_{r0}=-E_{i0},\ \theta_r=\theta_i$

$\vec{E}_r(x,z)=-\hat{y}E_{i0}e^{-j\beta_1(x\sin\theta_i-z\cos\theta_i)}$, $\vec{H}_r(x,z)=\frac{1}{\eta_1}[\hat{a}_{nr}\times\vec{E}_r(x,z)]=\frac{E_{i0}}{\eta_1}(-\hat{x}\cos\theta_i+\hat{z}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i-z\cos\theta_i)}$

Case 2 Parallel Polarization **(TM):**

$\hat{a}_{ni}=\hat{x}\sin\theta_i+\hat{z}\cos\theta_i$, $\hat{a}_{nr}=\hat{x}\sin\theta_r-\hat{z}\cos\theta_r$



Incident plane wave: $\vec{H}_i(x,z)=\hat{y}\frac{E_{i0}}{\eta_1}e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)}$

$$\vec{E}_i(x,z)=-\eta_1[\hat{a}_{ni}\times\vec{H}_i(x,z)]=E_{i0}(\hat{x}\cos\theta_i-\hat{z}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i+z\cos\theta_i)}$$

$\because\ \vec{E}_1(x,0)=\vec{E}_i(x,0)+\vec{E}_r(x,0)=0$, $\therefore$ $E_{r0}=-E_{i0},\ \theta_r=\theta_i$

$\vec{E}_r(x,z)=-E_{i0}(\hat{x}\cos\theta_i-\hat{z}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i-z\cos\theta_i)}$, $\vec{H}_r(x,z)=\hat{y}\frac{E_{i0}}{\eta_1}e^{-j\beta_1(x\sin\theta_i-z\cos\theta_i)}$

Eg. A uniform sinusoidal plane wave $E_i(x,y)=\hat{y}10e^{-j(6x+8z)}$ in air is incident on a perfectly conducting plane at *z*=0. (a) Find the frequency and wavelength of the wave. (b) Determine the incident angle. (c) Find $\vec{E}_r(x,z)$ and $\vec{H}_r(x,z)$ of the reflected wave.

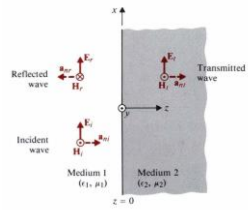
(Sol.) TE case:

(a) $\beta^2=6^2+8^2 \Rightarrow \beta=\dfrac{\omega}{v_p} \Rightarrow \omega=3\times10^9,\quad f=\dfrac{3\times10^9}{2\pi},\quad \lambda=\dfrac{\pi}{5}.$

(b) $\tan\theta_i=\dfrac{3}{4} \Rightarrow \theta_i=\tan^{-1}(\dfrac{3}{4})\approx 37°$

(c) $E_r=-10 \Rightarrow \vec{E}_r(x,z)=-\hat{y}10e^{-j(6x-8z)},\quad \vec{H}_r=\dfrac{1}{\eta_0}\hat{a}_{nr}\times\vec{E}_r(x,z)=-\dfrac{1}{12\pi}(\dfrac{\hat{x}4}{5}+\dfrac{\hat{z}3}{5})e^{-j(6x-8z)}$

3-3 Normal Incidence at a Plane Dielectric Boundary



Incident plane wave: $\begin{cases}\vec{E}_i(z)=\hat{x}E_{io}e^{-j\beta_1 z}\\ \vec{H}_i(z)=\hat{y}\dfrac{E_{io}}{\eta_1}e^{-j\beta_1 z}\end{cases}$, Reflected wave:

$\begin{cases}\vec{E}_r(z)=\hat{x}E_{ro}e^{j\beta_1 z}\\ \vec{H}_r(z)=-\hat{y}\dfrac{E_{ro}}{\eta_1}e^{j\beta_1 z}\end{cases}$ Transmitted wave: $\begin{cases}\vec{E}_t(z)=\hat{x}E_{to}e^{-j\beta_2 z}\\ \vec{H}_t(z)=\hat{y}\dfrac{E_{to}}{\eta_2}e^{-j\beta_2 z}\end{cases}$

Continuity of tangential field component at *z*=0

$$\Rightarrow \begin{cases}E_i(0)+E_r(0)=E_t(0)\\ H_i(0)+H_r(0)=H_t(0)\end{cases} \Rightarrow \begin{cases}E_{i0}+E_{r0}=E_{t0}\\ \dfrac{1}{\eta_1}(E_{i0}-E_{r0})=\dfrac{E_{t0}}{\eta_2}\end{cases} \Rightarrow \begin{cases}E_{r0}=\dfrac{\eta_2-\eta_1}{\eta_1+\eta_2}E_{i0}\\ E_{t0}=\dfrac{2\eta_2}{\eta_2+\eta_1}E_{i0}\end{cases}$$

Reflection coefficient: $\Gamma=\dfrac{E_{r0}}{E_{i0}}=-\dfrac{H_{r0}}{H_{i0}}=\dfrac{\eta_2-\eta_1}{\eta_2+\eta_1}$

Transmission coefficient: $\tau=\dfrac{E_{t0}}{E_{i0}}=\dfrac{2\eta_2}{\eta_2+\eta_1}$ and $1+\Gamma=\tau \Rightarrow \begin{cases}E_t(z)=\hat{x}\tau E_{i0}e^{-j\beta_2 z}\\ H_t(z)=\hat{y}\dfrac{\tau E_{i0}e^{-i\beta_2 z}}{\eta_2}\end{cases}$

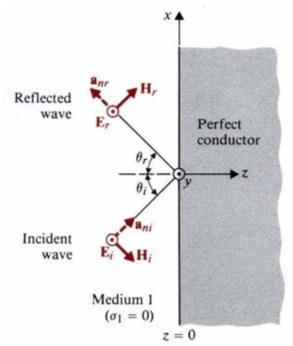
Standing wave ratio (*SWR*): $S=\dfrac{|E|_{\max}}{|E|_{\min}}=\dfrac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow |\Gamma|=\dfrac{S-1}{S+1}$

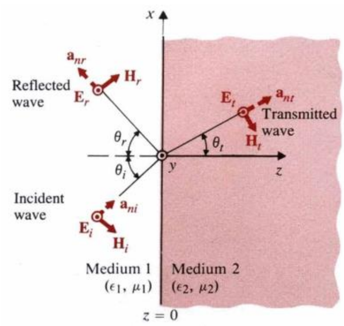
Eg. What condition $|\vec{E}_r| = |\vec{E}_t|$ occurs in normal incident case? And Standing wave ratio =?

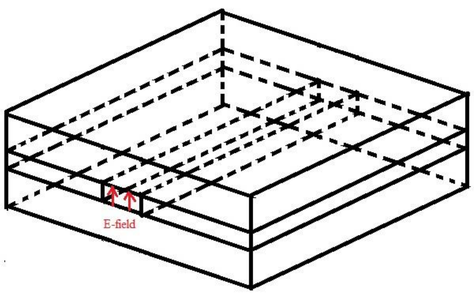
(Sol.) $\left|\dfrac{\eta_2-\eta_1}{\eta_2+\eta_1}\right| = \left|\dfrac{2\eta_2}{\eta_2+\eta_1}\right| \Rightarrow \begin{cases} \eta_1 = -\eta_2 (not\ reasonable) \\ \eta_1 = 3\eta_2 \Rightarrow \Gamma = -\dfrac{1}{2},\ |\Gamma| = \dfrac{1}{2} \end{cases}$, $S = \dfrac{1+|\Gamma|}{1-|\Gamma|} = 3$

Note: Definitions of TE and TM

TE:

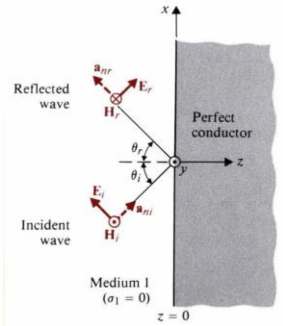


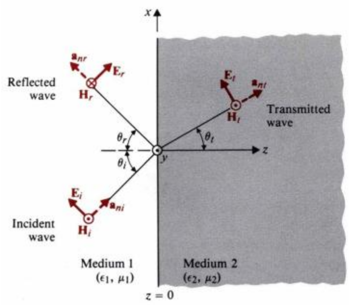


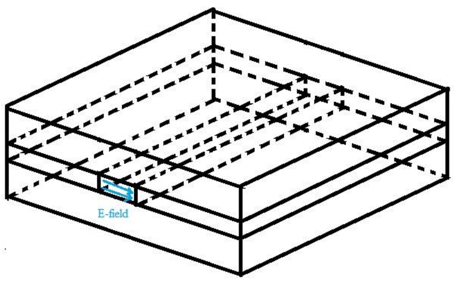


E_z=0

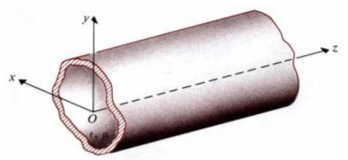
TM:



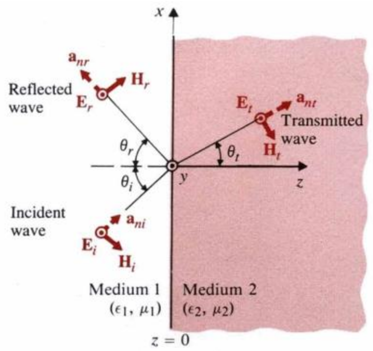




H_z=0



3-4 Oblique Incidences at a Plane Dielectric Boundary



Case 1 Perpendicular Polarization **(TE):**

$\hat{a}_{ni} = \hat{x}\sin\theta_i + \hat{z}\cos\theta_i$,

$\hat{a}_{nr} = \hat{x}\sin\theta_r - \hat{z}\cos\theta_r$,

$\hat{a}_{nt} = \hat{x}\sin\theta_t + \hat{z}\cos\theta_t$.

Incident plane wave:

$$\begin{cases} \vec{E}_i(x,z) = \hat{y}E_{io}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \\ \vec{H}_i(x,z) = \dfrac{E_{io}}{\eta_1}(-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \end{cases},$$

Reflected wave:

$$\begin{cases} \vec{E}_r(x,z) = \hat{y}E_{ro}e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \\ \vec{H}_r(x,z) = \dfrac{E_{ro}}{\eta_1}(\hat{x}\cos\theta_r + \hat{z}\sin\theta_r)e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \end{cases},$$

Transmitted (refractive) wave:

$$\begin{cases} \vec{E}_t(x,z) = \hat{y}E_{to}e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \\ \vec{H}_t(x,z) = \dfrac{E_{to}}{\eta_2}(-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \end{cases},$$

Continuity of tangential field components at $z=0$ $\Rightarrow$

$$\begin{cases} E_{iy}(x,0) + E_{ry}(x,0) = E_{ty}(x,0) \\ H_{ix}(x,0) + H_{rx}(x,0) = H_{tx}(x,0) \end{cases}$$

$$\Rightarrow \begin{cases} E_{i0}e^{-j\beta_1 x\sin\theta_i} + E_{r0}e^{-j\beta_1 x\sin\theta_i} = E_{t0}e^{-j\beta_2\sin\theta_t} \\ \dfrac{1}{\eta_1}(-E_{i0}\cos\theta_i e^{-j\beta_1 x\sin\theta_i} + E_{r0}\cos\theta_r e^{-j\beta_1 x\sin\theta_r}) = -\dfrac{E_{t0}}{\eta_2}\cos\theta_t e^{-j\beta_2 x\sin\theta_t} \end{cases}$$

$$\Rightarrow \begin{cases} \beta_1 x\sin\theta_i = \beta_1 x\sin\theta_r = \beta_2 x\sin\theta_t\ ,\ E_{i0} + E_{r0} = E_{t0} \\ \dfrac{1}{\eta_1}(E_{i0} - E_{r0})\cos\theta_i = \dfrac{E_{t0}}{\eta_2}\cos\theta_t \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{\beta_1}{\beta_2} = \dfrac{n_1}{n_2} = \dfrac{v_{p2}}{v_{p1}} = \dfrac{\eta_2}{\eta_1},\quad \theta_i = \theta_r \\ \Gamma_\perp = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_2/\cos\theta_t - \eta_1/\cos\theta_i}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i},\quad \tau_\perp = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} \end{cases}$$

and $1+\Gamma_\perp=\tau_\perp$.

Note: Snell's Law holds only in case of lossless media.

Brewster angle: No reflection occurs when $\sin\theta_{B\perp} = \sqrt{\dfrac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}} = \dfrac{1}{\sqrt{1+(\mu_1/\mu_2)}}$

if $\varepsilon_1=\varepsilon_2$, $\mu_1\neq\mu_2$

(Proof) $\Gamma_\perp = 0 \Rightarrow \eta_2 \cos\theta_i = \eta_1 \cos\theta_t$,

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_i} = \frac{\eta_2}{\eta_1}\cos\theta_i = \frac{\sqrt{\mu_1/\varepsilon_1}}{\sqrt{\mu_2/\varepsilon_2}}\cos\theta_i \Rightarrow \sin\theta_{B\perp} = \sqrt{\frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}}.$$

Eg. A uniform plane wave $\vec{E}_i(x,z) = \hat{y}6e^{-j(4x+3z)}$ in medium 1 ($4\varepsilon_0,\mu_0$) is incident on a plane of medium 2 ($64\varepsilon_0/9,\mu_0$) at z=0. (a) Is it in perpendicular polarization or parallel polarization? (b) Find the frequency and wavelength of the wave in medium 1. (c) Write down the reflected angle and the refractive angle of the transmitted wave. (d) Write down the unit propagation vectors of the incident wave $\hat{a}_{ni}$, the reflected wave $\hat{a}_{nr}$, and the transmitted wave $\hat{a}_{nt}$, respectively. (e) Compute the reflection and the transmission coefficients. (f) Find $\vec{E}_r(x,z)$ and $\vec{H}_r(x,z)$ of the reflected wave. (g) Find $\vec{E}_t(x,z)$ and $\vec{H}_t(x,z)$ of the transmitted wave.

(Sol.) (a) $\because \hat{y} \perp xz-plane$, $\therefore$ Perpendicular polarization (TE)

(b) $\beta_1 = \sqrt{4^2+3^2} = 5 = \dfrac{2\pi}{\lambda} = \omega\sqrt{\mu_0 4\varepsilon_0} = \dfrac{\omega 2}{c} \Rightarrow \lambda = \dfrac{2\pi}{5}$, $f = \dfrac{15}{4\pi}\times 10^8$

(c) $\dfrac{\sin\theta_i}{\sin\theta_t} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \dfrac{4}{3} \Rightarrow \sin\theta_t = \dfrac{3}{4}\sin\theta_i = \dfrac{3}{5}$, $\cos\theta_t = \dfrac{4}{5}$

(d) $\hat{a}_{ni} = \dfrac{4\hat{x}+3\hat{z}}{5}$, $\hat{a}_{nr} = \dfrac{4\hat{x}-3\hat{z}}{5}$, $\hat{a}_{nt} = \dfrac{3\hat{x}+4\hat{z}}{5}$

(e) $\eta_1 = \sqrt{\dfrac{\mu_0}{4\varepsilon_0}} = 60\pi$, $\eta_2 = \sqrt{\dfrac{\mu_0}{(64\varepsilon_0/9)}} = 45\pi$

$$\Gamma_\perp = \frac{\eta_2/\cos\theta_t - \eta_1/\cos\theta_i}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} = -\frac{7}{25}, \quad \tau_\perp = \frac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} = \frac{18}{25}$$

(f) $\vec{E}_r(x,z) = \Gamma_\perp \cdot \hat{y}6e^{-j(4x-3z)} = -\hat{y}\dfrac{42}{25}e^{-j(4x-3z)}$

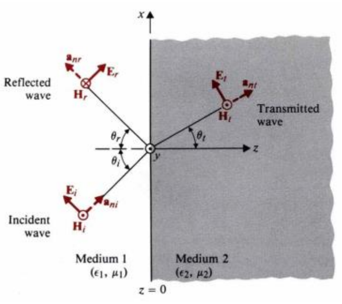
$$\vec{H}_r(x,z) = \frac{1}{\eta_1}(\hat{a}_{nr}\times\vec{E}_r) = \frac{-7}{1250\pi}(3\hat{x}+4\hat{z})e^{-j(4x-3z)}$$

(g) In medium 2, $\beta_2 = \dfrac{15}{2}\times 10^8 \cdot \sqrt{\mu_0 \cdot \dfrac{64}{9}\varepsilon_0} = \dfrac{20}{3}$

$$\vec{E}_t(x,z) = \tau_\perp \cdot \hat{y}6e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} = \hat{y}\frac{108}{25}e^{-j(4x+\frac{16}{3}z)}$$

$$\vec{H}_t(x,z) = \frac{1}{\eta_2}(\hat{a}_{nt}\times\vec{E}_t) = \frac{27}{125\pi}(-\frac{4}{5}\hat{x}+\frac{3}{5}\hat{z})e^{-j(4x+\frac{16}{3}z)}$$

***Case 2 Parallel Polarization* (TM):**



$\hat{a}_{ni} = \hat{x}\sin\theta_i + \hat{z}\cos\theta_i$, $\hat{a}_{nr} = \hat{x}\sin\theta_r - \hat{z}\cos\theta_r$

$\hat{a}_{nt} = \hat{x}\sin\theta_t + \hat{z}\cos\theta_t$

Incident plane wave:

$$\begin{cases} \vec{H}_i(x,z) = \hat{y}\dfrac{E_{i0}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \\ \vec{E}_i(x,z) = E_{i0}(\hat{x}\cos\theta_i - \hat{z}\sin\theta_i) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \end{cases},$$

Reflected wave:
$$\begin{cases} \vec{E}_r(x,z) = E_{ro}(\hat{x}\cos\theta_r + \hat{z}\sin\theta_r) e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \\ \vec{H}_r(x,z) = -\hat{y}\dfrac{E_{r0}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \end{cases},$$

Transmitted wave:
$$\begin{cases} \vec{E}_t(x,z) = E_{to}(\hat{x}\cos\theta_t - \hat{z}\sin\theta_t) e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \\ \vec{H}_t(x,z) = \hat{y}\dfrac{E_{t0}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \end{cases}.$$

Continuity of tangential field components at z=0 $\Rightarrow \dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{\beta_1}{\beta_2} = \dfrac{v_{p2}}{v_{p1}} = \dfrac{\eta_2}{\eta_1}$

$$\begin{cases} (E_{i0} + E_{r0})\cos\theta_i = E_{t0}\cos\theta_t \\ \dfrac{1}{\eta_1}(E_{i0} - E_{r0}) = \dfrac{E_{t0}}{\eta_2} \end{cases} \Rightarrow \begin{cases} \Gamma_{\parallel} = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \\ \tau_{\parallel} = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \end{cases}$$, and $1 + \Gamma_{\parallel} = \tau_{\parallel} \cdot \dfrac{\cos\theta_t}{\cos\theta_i}$

Brewster angle: No reflection occurs when $\sin\theta_{B\parallel} = \sqrt{\dfrac{1 - \mu_2\varepsilon_1/\mu_1\varepsilon_2}{1 - (\varepsilon_1/\varepsilon_2)^2}} = \dfrac{1}{\sqrt{1 + (\varepsilon_1/\varepsilon_2)}}$

If $\mu_1 = \mu_2 \Rightarrow \theta_{B\parallel} = \tan^{-1}\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \tan^{-1}(\dfrac{n_1}{n_2})$.

Eg. A uniform plane wave $\vec{E}_i(x,z)=(-3\hat{x}+4\hat{z})e^{-j(8x+6z)}$ in air is incident on a plane of medium ($16\varepsilon_0/9,\mu_0$) at z=0. (a) Is it in perpendicular polarization or parallel polarization? (b) Find the frequency and wavelength of the wave in medium 1. (c) Write down the refractive angle θ_t of the transmitted wave. Is θ_i equal to the Brewster angle? (d) Write down the unit propagation vectors of the incident wave $\hat{a}_{ni}$, the reflected wave $\hat{a}_{nr}$, and the transmitted wave $\hat{a}_{nt}$, respectively. (e) Compute the reflection and the transmission coefficients. (f) Find $\vec{E}_r(x,z)$ and $\vec{H}_r(x,z)$ of the reflected wave. (g) Find $\vec{E}_t(x,z)$ and $\vec{H}_t(x,z)$ of the transmitted wave.

(Sol.) (a) $\vec{H}_i=\dfrac{1}{\eta_0}(\hat{a}_{ni}\times\vec{E})=-\hat{y}5e^{-j(8x+6z)}$, $\therefore$ Parallel polarization (TM)

(b) $\beta_1=\sqrt{8^2+6^2}=10=\dfrac{2\pi}{\lambda}=\omega\sqrt{\mu_0\varepsilon_0}\Rightarrow\lambda=\dfrac{\pi}{5}m$

$\omega=3\times10^9\Rightarrow f=\dfrac{15}{\pi}\times10^8$

(c) $\theta_i=\sin^{-1}(\dfrac{4}{5})$, $\sin\theta_B=\dfrac{1}{\sqrt{1+(\dfrac{\varepsilon_1}{\varepsilon_2})}}=\dfrac{4}{5}$, $\therefore\theta_i=\theta_B$

$$\frac{\sin\theta_i}{\sin\theta_t}=\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}=\frac{4}{3}\Rightarrow\sin\theta_t=\frac{4}{5}\times\frac{3}{4}=\frac{3}{5}\Rightarrow\theta_t=\sin^{-1}(\frac{3}{5}),\quad\cos\theta_t=\frac{4}{5}$$

(d) $\hat{a}_{ni}=\dfrac{4\hat{x}+3\hat{z}}{5}$, $\hat{a}_{nr}=0$, $\hat{a}_{nt}=\dfrac{3\hat{x}+4\hat{z}}{5}$

(e) $\eta_1=\sqrt{\dfrac{\mu_0}{\varepsilon_0}}=120\pi$, $\eta_2=\sqrt{\dfrac{\mu_0}{(16\varepsilon_0/9)}}=90\pi\Rightarrow\Gamma_{\|}=\dfrac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}=0$,

$$\tau_{\|}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}=\frac{3}{4}$$

(f) $\vec{E}_r(x,z)=\vec{H}_r(x,z)=0$

(g) In medium 2, $\beta_2=\omega\sqrt{\mu_0\cdot\dfrac{16}{9}\varepsilon_0}=\dfrac{40}{3}$

$$\vec{E}_t(x,z)=\tau_{\|}\cdot(-3\hat{x}+4\hat{z})e^{-j\beta_2(x\sin\theta_t+z\cos\theta_t)}=(-\frac{9}{4}\hat{x}+3\hat{z})e^{-j(8x+\frac{32}{3}z)}$$

$$\vec{H}_t(x,z)=\frac{1}{\eta_2}(\hat{a}_{nt}\times\vec{E}_t)=\frac{1}{90\pi}(-\frac{18}{5}\hat{y})e^{-j(8x+\frac{32}{3}z)}=-\hat{y}\frac{1}{25\pi}e^{-j(8x+\frac{32}{3}z)}$$

Snell's Law: $\frac{\sin\theta_t}{\sin\theta_i}=\frac{\beta_1}{\beta_2}=\frac{n_1}{n_2}=\frac{v_{p2}}{v_{p1}}=\frac{\eta_2}{\eta_1}=\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}=\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$ and $\theta_r=\theta_i$

The reflection coefficient: $\Gamma=\frac{E_{r0}}{E_{i0}}$ and **the transmission coefficient:** $\tau=\frac{E_{t0}}{E_{i0}}$

$$\begin{cases}\Gamma_\perp=\dfrac{\eta_2/\cos\theta_t-\eta_1/\cos\theta_i}{\eta_2/\cos\theta_t+\eta_1/\cos\theta_i}=\dfrac{n_1\cos\theta_i-n_2\cos\theta_t}{n_1\cos\theta_i+n_2\cos\theta_t}=\dfrac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)}\\ \tau_\perp=\dfrac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t+\eta_1/\cos\theta_i}=\dfrac{2n_1\cos\theta_i}{n_1\cos\theta_i+n_2\cos\theta_t}=\dfrac{2\cos\theta_i\sin\theta_t}{\sin(\theta_t+\theta_i)}\end{cases}$$

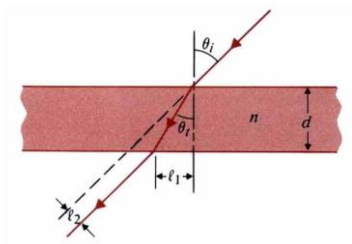
for perpendicular polarization (TE)

$$\begin{cases}\Gamma_\parallel=\dfrac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}=\dfrac{n_1/\cos\theta_i-n_2/\cos\theta_t}{n_1/\cos\theta_i+n_2/\cos\theta_t}=\dfrac{\tan(\theta_t-\theta_i)}{\tan(\theta_t+\theta_i)}\\ \tau_\parallel=\dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i}=\dfrac{2n_1/\cos\theta_t}{n_1/\cos\theta_i+n_2/\cos\theta_t}=\dfrac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}\end{cases}$$

for parallel polarization (TM)

Eg. A light ray is incident from air obliquely on a transparent sheet of thickness *d* with an index of refraction as shown in the figure. The angle of incidence is θ_i. Find (a) θ_t, (b) the distance l_1 at the point of exit, and (c) the amount of the lateral displacement l_2 of the emerging ray.

(Sol.) (a) $\frac{\sin\theta_t}{\sin\theta_i}=\frac{1}{n}\Rightarrow\theta_t=\sin^{-1}(\frac{\sin\theta_i}{n})$



(b) $\ell_1=d\tan\theta_t=\frac{d\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}$

(c)

$$\ell_2=\sqrt{\ell_1^2+d^2}\cdot\sin(\theta_i-\theta_t)=\sqrt{\frac{d^2\sin^2\theta_i}{n^2-\sin^2\theta_i}+d^2}\cdot[\sin\theta_i\cos\theta_t-\cos\theta_i\sin\theta_t]=d\sin\theta_i-\frac{d\sin\theta_i\cos\theta_i}{\sqrt{n^2-\sin^2\theta_i}}$$

Rigid Body (sings).

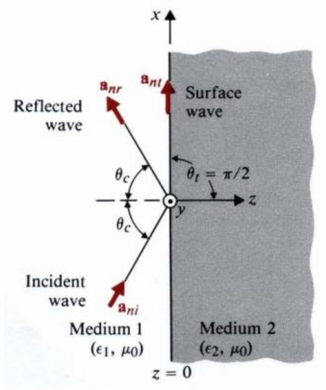
Gin a body meet a body
Flyin' through the air,
Gin a body hit a body,
Will it fly? and where?
Ilka impact has its measure,
Ne'er a ane hae I,
Yet a' the lads they measure me,
Or, at least, they try.

Gin a body meet a body
Altogether free,
How they travel afterwards
We do not always see.
Ilka problem has its method
By analytics high;
For me, I ken na ane o' them,
But what the waur am I?

by James Clerk Maxwell

3-5 Total Reflection and Critical Angle (θ_c)

In case of $\varepsilon_1 > \varepsilon_2$ (or $n_1 > n_2$):



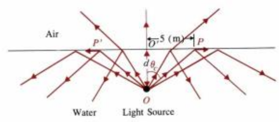
Define critical angle as $\theta_c \equiv \sin^{-1}(\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}}) = \sin^{-1}(\dfrac{n_2}{n_1})$,

While $\theta_i > \theta_c \Rightarrow \sin\theta_t = \sqrt{\dfrac{\varepsilon_1}{\varepsilon_2}}\sin\theta_i > 1$

$$\Rightarrow \cos\theta_t = \sqrt{1-\sin^2\theta_t} = \pm j\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1}$$

$\hat{a}_{nt} = \hat{x}\sin\theta_t + \hat{z}\cos\theta_t$, $\vec{E}_t$ and $\vec{H}_t \propto e^{-j\beta_2 z\cos\theta_t}\cdot e^{-j\beta_2 x\sin\theta_t} = e^{-\alpha_2 z}\cdot e^{-j\beta_{2x}x} \to 0$ as $z\to\infty$, where $\alpha_2 = \beta_2\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}\sin^2\theta_t - 1}$ and $\beta_{2x} = \beta_2\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}}\sin\theta_i$

Eg. ε of water at optical frequency $=1.75\varepsilon_0$, find d at a distance under water yields an illuminated circular area of a radius r=5m.

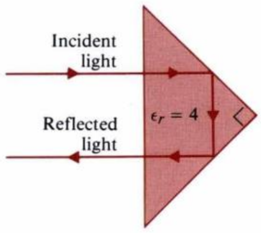


(Sol.) $\theta_c = \sin^{-1}\sqrt{\dfrac{1}{1.75}} = 49.2°$, $d\tan 49.2° = 5 \Rightarrow d = 4.32m$

Eg. Assuming ε_r=4 for glass, calculate the percentage of the incident light power reflected back by the prism.

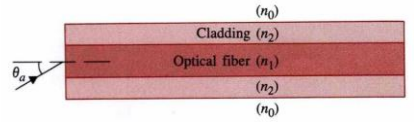
(Sol.) $\theta_c = \sin^{-1}\sqrt{\dfrac{1}{4}} = 30° < 45°$, $\eta_{air} = 120\pi$, $\eta_{prism} = 60\pi$



$$\tau_{air-prism} = \frac{2\times 60\pi}{120\pi + 60\pi} = \frac{2}{3},\quad \tau_{prism-air} = \frac{2\times 120\pi}{120\pi + 60\pi} = \frac{4}{3}$$

$$\tau_{total} = \left|\frac{2}{3}\right|^2 \cdot 1^2 \cdot 1^2 \cdot \left|\frac{4}{3}\right|^2 = \frac{64}{81} \approx 79\%$$

Eg. For preventing interference of waves in neighboring fibers and for mechanical protection, individual optical fibers are usually cladded by a material of a lower refractive index, as shown in the figure, where $n_1 > n_2$. (a) Express the maximum angle of incident θ_a in terms of n_0, n_1 and n_2 for meridional rays incident on the core's end face to be trapped inside the core by total internal reflection. (Meridional rays are those that pass through the fiber axis. The angle θ_a is called the acceptance angle, $\sin(\theta_a)$ is the numerical aperture (NA) of the fiber.)



(Sol.) $\sin\phi = \frac{n_0}{n_1}\sin\theta_a$, $\varphi = \frac{\pi}{2} - \phi > \theta_c = \sin^{-1}(\frac{n_2}{n_1})$

$$\sin\theta_c = \frac{n_2}{n_1} < \sin\varphi = \cos\phi = \sqrt{1 - \frac{n_0^{\,2}\sin^2\theta_a}{n_1^{\,2}}}$$

$$\frac{n_2^{\,2}}{n_1^{\,2}} < 1 - \frac{n_0^{\,2}\sin^2\theta_a}{n_1^{\,2}}\ ,\quad \sin\theta_a < \frac{\sqrt{n_1^{\,2} - n_2^{\,2}}}{n_0}$$, where $n_0=1$

Note: The **depth of focus (DOF)** of an optical imaging system is usually defined as **DOF=0.5λ^2/NA^2**.

3-6 Introduction to *S*-parameters

***S*-parameters:** $[S]=\begin{bmatrix} S_{11} \; S_{12} \\ S_{21} \; S_{22} \end{bmatrix}$ for analyzing the two-port high-frequency circuits.

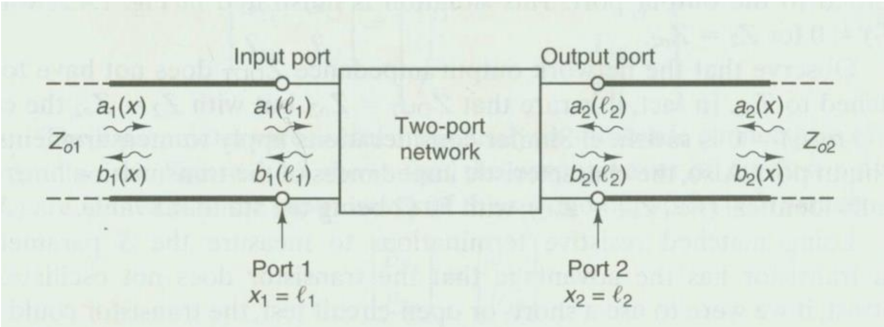


Figure Incident and reflected waves in a two-port network.

Define $a(x)=\dfrac{1}{2\sqrt{Z_0}}[V(x)+Z_0 I(x)]$, $b(x)=\dfrac{1}{2\sqrt{Z_0}}[V(x)-Z_0 I(x)]$

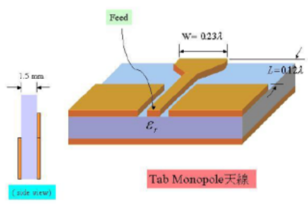
$$b_1(l_1)=S_{11}a_1(l_1)+S_{12}a_2(l_2),\ \ b_2(l_2)=S_{21}a_1(l_1)+S_{22}a_2(l_2)$$

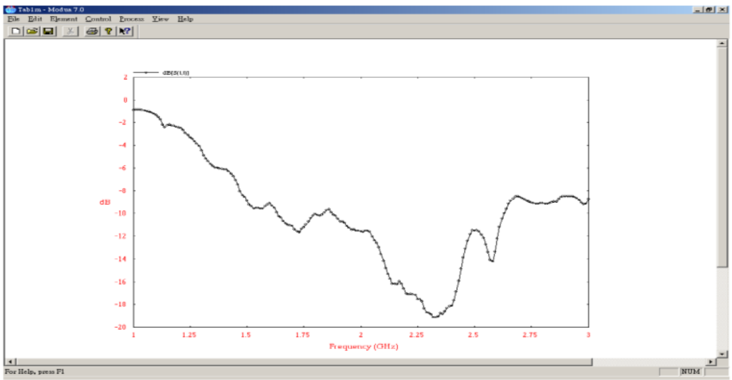
$$\Rightarrow \begin{bmatrix} b_1(l_1) \\ b_2(l_2) \end{bmatrix} = \begin{bmatrix} S_{11} S_{12} \\ S_{21} S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1(l_1) \\ a_2(l_2) \end{bmatrix},$$

where $S_{11}=\dfrac{b_1(l_1)}{a_1(l_1)}\Big|_{a_2(l_2)=0}$, $S_{21}=\dfrac{b_2(l_2)}{a_1(l_1)}\Big|_{a_2(l_2)=0}$, $S_{22}=\dfrac{b_2(l_2)}{a_2(l_2)}\Big|_{a_1(l_1)=0}$, and

$$S_{12}=\frac{b_1(l_1)}{a_2(l_2)}\Big|_{a_1(l_1)=0}.$$

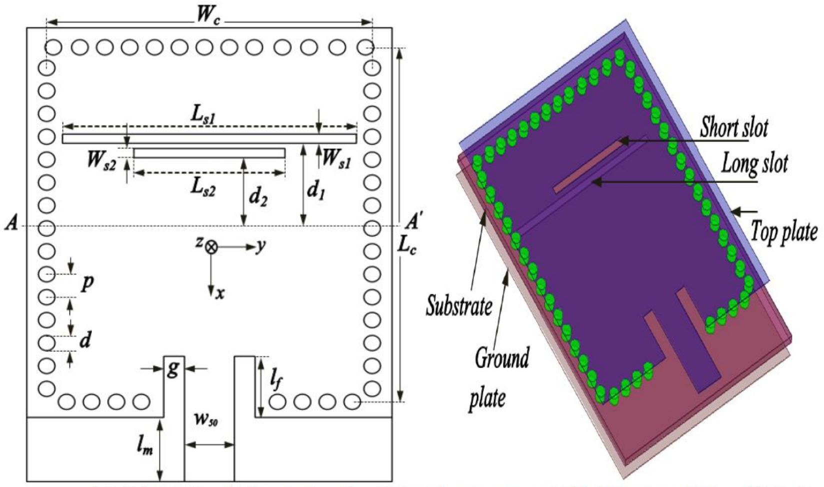
Eg. The variation of S_{11} parameter of a tab monopole antenna versus operating frequency.



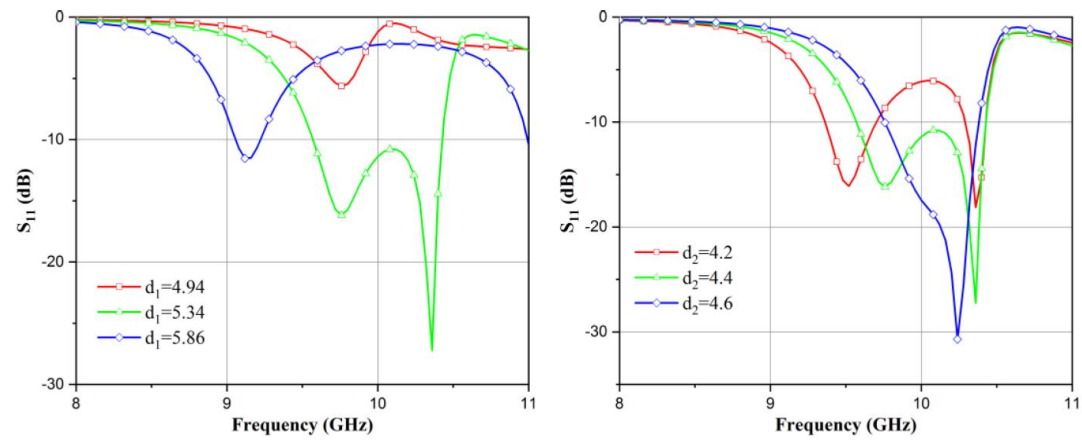


The S_{11} parameter of the Tab Monopole

Eg. The variation of S_{11} parameter of a wideband low-profile SIW cavity-backed bilateral slots antenna for X-band versus operating frequency.

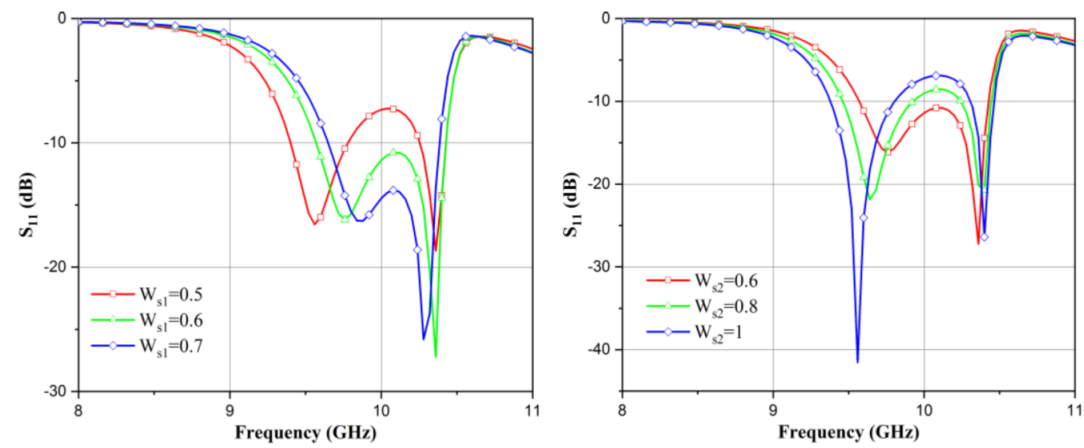


(a) Schematic of the designed wideband antenna and (b) 3D view. [W_c = 19.6, L_c = 23.2, L_{s1} = 17.7, W_{s1} = 0.6, d_1 = 5.34, L_{s2} = 9.1, W_{s2} = 0.6, d_2 = 4.4, p = 1.5, d = 1, w_{50} = 3, l_f = 4, g = 1.25, l_m = 4.2, h = 0.5]. (Units: mm)



(a) (b)

Reflection coefficient (S_{11}) variations with different values of (a) d_1 (b) d_2.

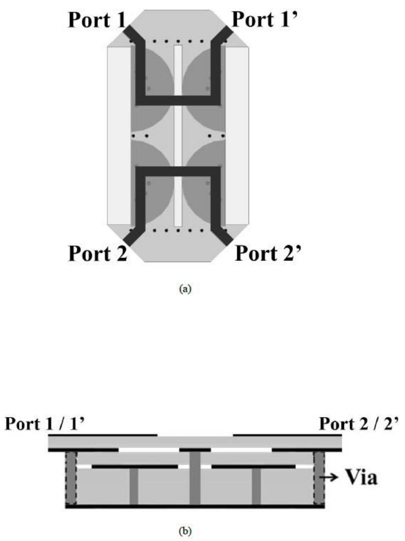


(a) (b)

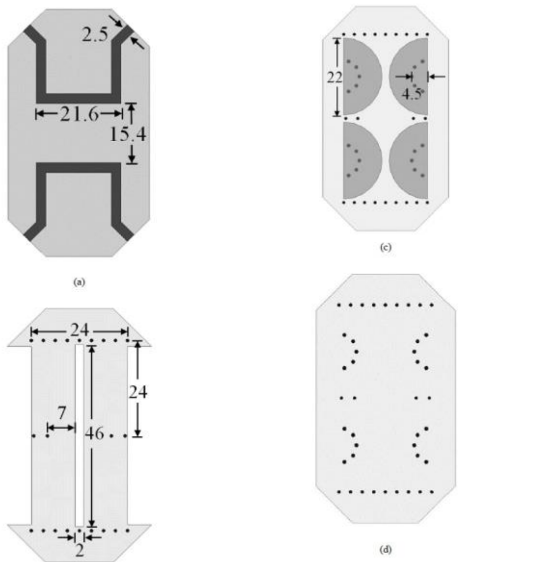
Reflection coefficient (S_{11}) variations by slot width (a) W_{s1} (b) W_{s2}.

Eg. Compact balanced bandpass filter using miniaturized substrate integrated waveguide cavities.

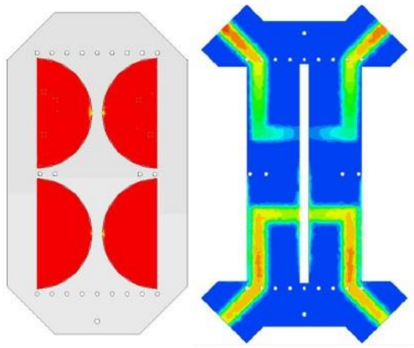
半模態之平衡式縮小濾波器實作圖



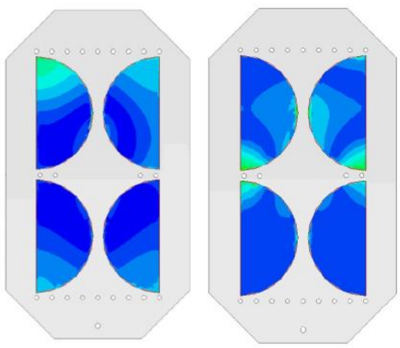
半模態平衡式之縮小化濾波器結構(a)示意圖(b)側視圖



半模態平衡式之縮小化濾波器俯視圖(a)第一層(b)第二層(c)第三層(d) 第四層

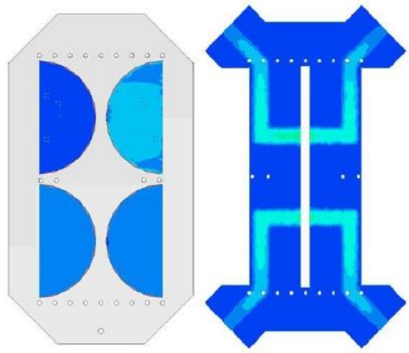


(a)



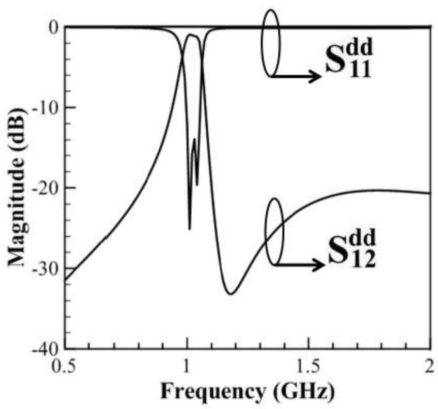
(a) (b)

需求頻帶外共振點之高階模態腔體內電場分布情形(a) 3.19 GHz (b) 5.30 GHz

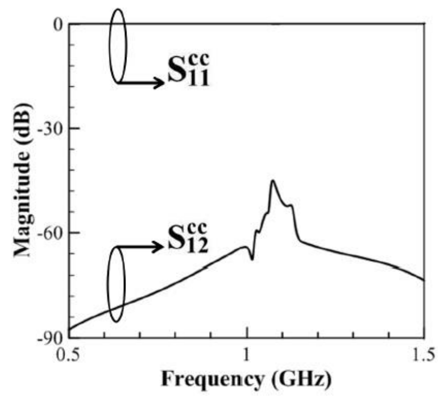


(b)

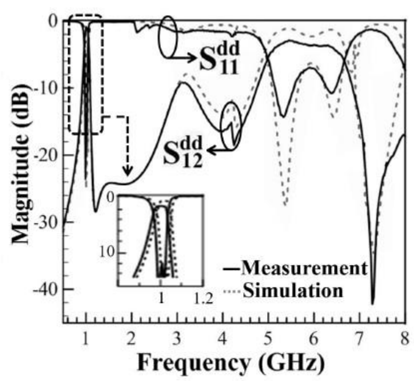
通帶內之電場分布(a)差模響應電場圖(b)共模響應電場圖



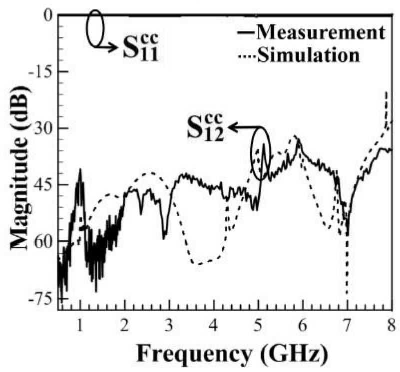
(a)



(b)



(c)



(d)

半模態平衡式縮小濾波器之響應圖(a)差模模擬(b)共模模擬
(c)差模模擬與實測之寬頻響應(d)共模模擬與實測之寬頻響應

New *S*-parameters obtained by shifting reference planes:

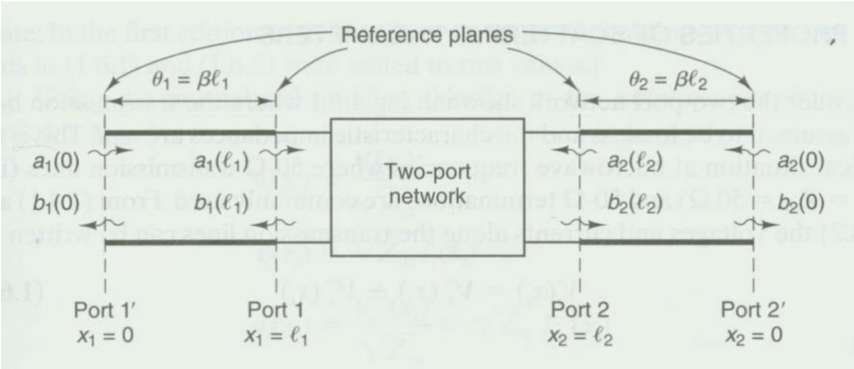


Figure Model for shifting reference planes.

$$b_1(l_1) = b_1(0)e^{j\theta_1},\quad a_1(l_1) = a_1(0)e^{-j\theta_1},\quad b_2(l_2) = b_2(0)e^{j\theta_2},\quad a_2(l_2) = a_2(0)e^{-j\theta_2}$$

$$\Rightarrow \begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix} \cdot \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix}, \text{ where}$$

$$\begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix} \text{ and } \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} S'_{11}e^{j2\theta_1} & S'_{12}e^{j(\theta_1+\theta_2)} \\ S'_{21}e^{j(\theta_1+\theta_2)} & S'_{22}e^{j2\theta_2} \end{bmatrix}$$

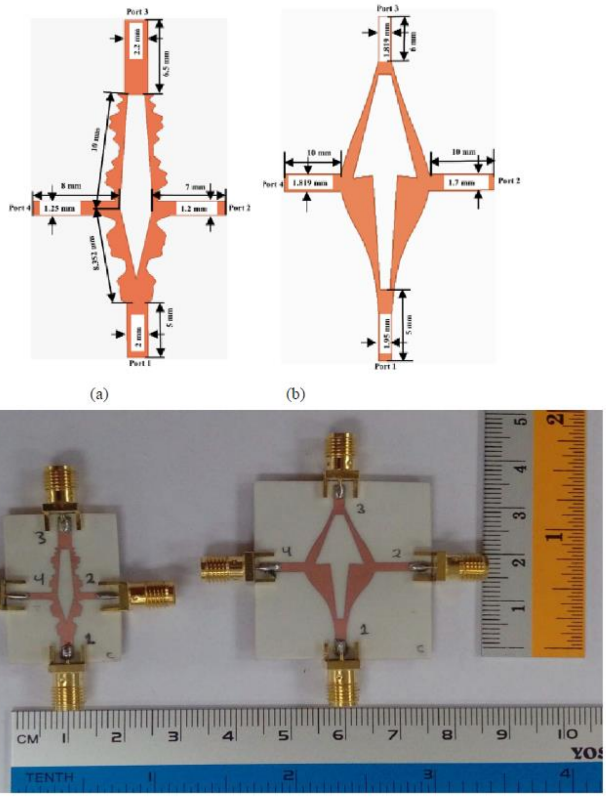
***T*-parameters:** $\begin{bmatrix} a_1(l_1) \\ b_1(l_1) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{bmatrix} b_2(l_2) \\ a_2(l_2) \end{bmatrix}$, where $\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{S_{21}} & -\dfrac{S_{22}}{S_{21}} \\ \dfrac{S_{11}}{S_{21}} & S_{12} - \dfrac{S_{11}S_{22}}{S_{21}} \end{bmatrix}$

and $\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{T_{21}}{T_{11}} & T_{22} - \dfrac{T_{21}T_{12}}{T_{11}} \\ \dfrac{1}{T_{11}} & -\dfrac{T_{12}}{T_{11}} \end{bmatrix}$

For analyzing three-port, four-port, …, *n*-port high-frequency circuits, the *S*-parameters are expressed in the following ways:

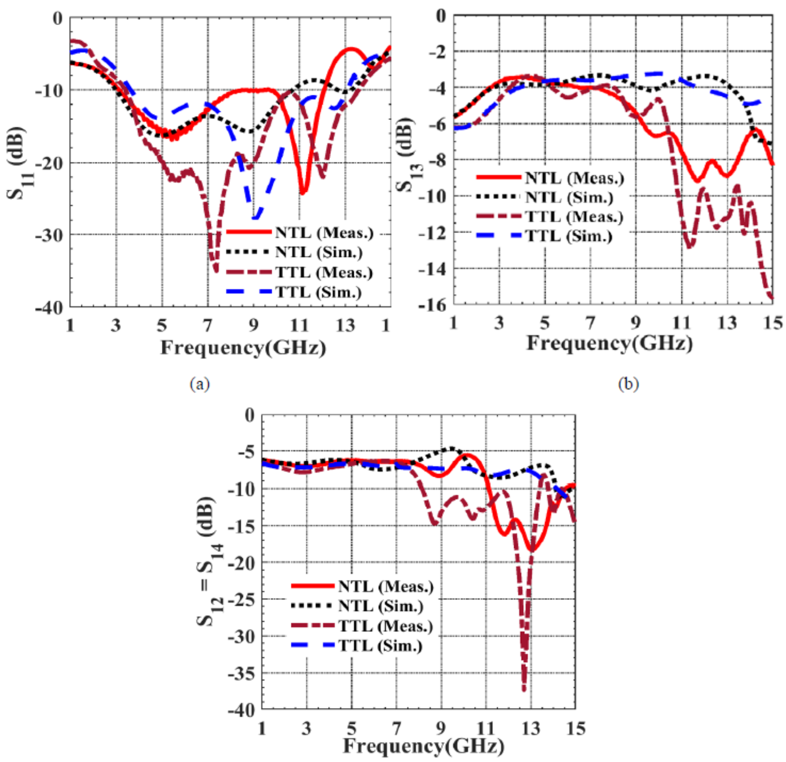
$$\begin{bmatrix} b_1(l_1) \\ b_2(l_2) \\ b_3(l_3) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \cdot \begin{bmatrix} a_1(l_1) \\ a_2(l_2) \\ a_3(l_3) \end{bmatrix},\quad \begin{bmatrix} b_1(l_1) \\ b_2(l_2) \\ b_3(l_3) \\ b_4(l_4) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \cdot \begin{bmatrix} a_1(l_1) \\ a_2(l_2) \\ a_3(l_3) \\ a_4(l_4) \end{bmatrix}, \ldots$$

Eg. The variation of S-parameters of a compact UWB 1:2:1 unequal-split 3-way Bagley power divider using non-uniform transmission lines.



(a) (b) (c)

Configuration of the proposed compact **(a)** NTL layout **(b)** TTL UWB 1:2:1unequal split 3-way BPD layout and **(c)** fabricated prototypes.

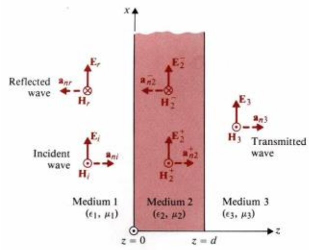


(a) (b) (c)

S-Parameters **(a)** S_{11}, **(b)**S_{12} =S_{14} and **(c)** S_{13} of NTL and TTL UWB 1:2:1unequal split 3-way

3-7 Normal Incidence at Three-layer Dielectric Interfaces

In medium 1,



$$\begin{cases} \vec{E}_1 = \hat{x}(E_{i0}e^{-j\beta_1 z} + E_{r0}e^{j\beta_1 z}) \\ \vec{H}_1 = \hat{y}\dfrac{1}{\eta_1}(E_{i0}e^{-j\beta_1 z} - E_{r0}e^{j\beta_1 z}) \end{cases}, \quad E_{r0} = \Gamma_0 \cdot E_{i0}$$

In medium 2,

$$\begin{cases} \vec{E}_2 = \hat{x}(E_2{}^{+}e^{-j\beta_2 z} + E_2{}^{-}e^{j\beta_2 z}) \\ \vec{H}_2 = \hat{y}\dfrac{1}{\eta_2}(E_2{}^{+}e^{-j\beta_2 z} - E_2{}^{-}e^{j\beta_2 z}) \end{cases}$$

In medium 3,

$$\begin{cases} \vec{E}_3 = \hat{x}E_3{}^{+}e^{-j\beta_3 z} \\ \vec{H}_3 = \hat{y}\dfrac{E_3{}^{+}}{\eta_3}e^{-j\beta_3 z} \end{cases}$$

∵ Continuity of tangential field components at z=0 & z=d,

∴ $E_1(0)=E_2(0)$, $H_1(0)=H_2(0)$, $E_2(d)=E_3(d)$, $H_2(d)=H_3(d)$

Define $Z_2(0) = \eta_2 \cdot \dfrac{\eta_3 \cos\beta_2 d + j\eta_2 \sin\beta_2 d}{\eta_2 \cos\beta_2 d + j\eta_3 \sin\beta_2 d} = \eta_2 \cdot \dfrac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d}$

$\Rightarrow$ **Effective reflection coefficient Γ_0=** $\dfrac{E_{r0}}{E_{i0}} = -\dfrac{H_{r0}}{H_{i0}} = \dfrac{Z_2(0)-\eta_1}{Z_2(0)+\eta_1}$

Eg. If no reflection occurs, find the relation among d, η_1, η_2, and η_3.

(Sol.)

$$\Gamma_0 = 0 = \frac{Z_2(0)-\eta_1}{Z_2(0)+\eta_1} \Rightarrow \eta_2(\eta_3\cos\beta_2 d + j\eta_2 \sin\beta_2 d) = \eta_1\eta_2\cos\beta_2 d + j\eta_3 \sin\beta_2 d)$$

$$\Rightarrow \begin{cases} \eta_3 \cos\beta_2 d = \eta_1 \cos\beta_2 d \\ \eta_2{}^2 \sin\beta_2 d = \eta_1\eta_3 \sin\beta_2 d \end{cases} \Rightarrow \begin{cases} \eta_2 = \sqrt{\eta_1\eta_3} \\ \cos\beta_2 d = 0 \end{cases} \; or \; \begin{cases} \eta_1 = \eta_3 \\ \sin\beta_2 d = 0 \end{cases}$$

Case 1:

$$\begin{cases} \eta_2 = \sqrt{\eta_1\eta_3} \;\; or \;\; n_2 = \sqrt{n_1 n_3} \\ d = \dfrac{(2n+1)}{4}\lambda_2, \;\; n = 0,1,2,... \end{cases}$$

(Quarter-wave impedance transformer)

Case 2:

$$\begin{cases} \eta_1 = \eta_3 \;\; or \;\; n_1 = n_3 \\ d = \dfrac{n\lambda_2}{2}, \;\; n = 0,1,2,... \end{cases}$$

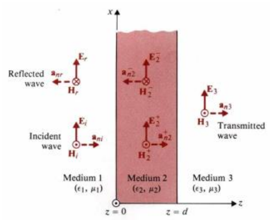
(Half-wave impedance transformer)

Eg. A transparent dielectric coating is applied to glass (μ_r=1, ε_r=4) to eliminate the reflection of red light λ_0=0.75μm. Determine the required dielectric constant and thickness of the coating.

(Sol.) (a) $\eta_1 = \eta_0 = 120\pi$, $\eta_3 = 60\pi$

$\eta_2 = \sqrt{\eta_1\eta_3} = \dfrac{120\pi}{\sqrt{2}} = \dfrac{120\pi}{\sqrt{\varepsilon_r}} \Rightarrow \varepsilon_r = 2 \Rightarrow n_2 = \sqrt{2}$, d=$\dfrac{(2n+1)}{4}\lambda_2 = \dfrac{(2n+1)}{4}\cdot\dfrac{0.75}{\sqrt{2}}$, n=0,1,2,…

3-8 Optical Theory of Multi-Layer Films



Normal incidence: At $z=0$: $\begin{cases} E_i + E_r = E_2 + E_2' \\ n_1 E_i - n_1 E_r = n_2 E_2 - n_2 E_2' \end{cases}$, $z=d$:

$$\begin{cases} E_2 e^{-jk_2 d} + E_2' e^{jk_2 d} = E_3 \\ n_2 E_2 e^{-jk_2 d} - n_2 E_2' e^{jk_2 d} = n_3 E_3 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 \\ n_1 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_1 \end{bmatrix} \cdot \left(\frac{E_r}{E_i}\right) = \begin{bmatrix} \cos(k_2 d) & \frac{-j\sin(k_2 d)}{n_2} \\ -jn_2 \sin(k_2 d) & \cos(k_2 d) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ n_3 \end{bmatrix} \cdot \left(\frac{E_3}{E_i}\right)$$

Or, $\begin{bmatrix} 1 \\ n_1 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_1 \end{bmatrix} \cdot \Gamma = M \cdot \begin{bmatrix} 1 \\ n_3 \end{bmatrix} \cdot \tau$, where $k_2=2\pi n_2/\lambda$, $\Gamma=E_r/E_i$, $\tau=E_3/E_i$

Optical transfer matrix: $M = \begin{bmatrix} \cos(k_2 d) & \frac{-j\sin(k_2 d)}{n_2} \\ -jn_2 \sin(k_2 d) & \cos(k_2 d) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$

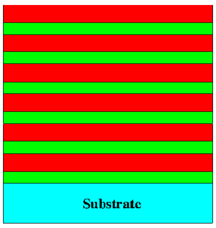
$$\Rightarrow \Gamma = \frac{M_{11}n_1 + M_{12}n_3 n_1 - M_{21} - M_{22}n_3}{M_{11}n_1 + M_{12}n_3 n_1 + M_{21} + M_{22}n_3} \quad \text{and} \quad \tau = \frac{2n_1}{M_{11}n_1 + M_{12}n_3 n_1 + M_{21} + M_{22}n_3}$$

Oblique incidence: $M=\begin{bmatrix} \cos[k_2 d\cos(\theta_i)] & \frac{-j\sin[k_2 d\cos(\theta_i)]}{p} \\ -jp\sin[k_2 d\cos(\theta_i)] & \cos[k_2 d\cos(\theta_i)] \end{bmatrix}$, where $p=n_2\cos\theta_i$ (TE), or $p=n_2/\cos\theta_i$ (TM)

Anti-reflecting film: $\Gamma=0$ and $d=\lambda/4n_2 \Rightarrow M_{11}=M_{22}=0$, $M_{12}=-j/n_2$, $M_{21}=-jn_2$

$\Rightarrow M_{12}n_3n_1=-jn_3n_1/n_2=M_{21}=-jn_2 \Rightarrow n_2=\sqrt{n_1 n_3}$

Effective optical transfer matrix of *n*-layer film: $M_t=M_1M_2M_3\ldots M_n$.



High-reflectance film: A stack of N alternate quarter-wave layers of high index n_h and low index n_l materials

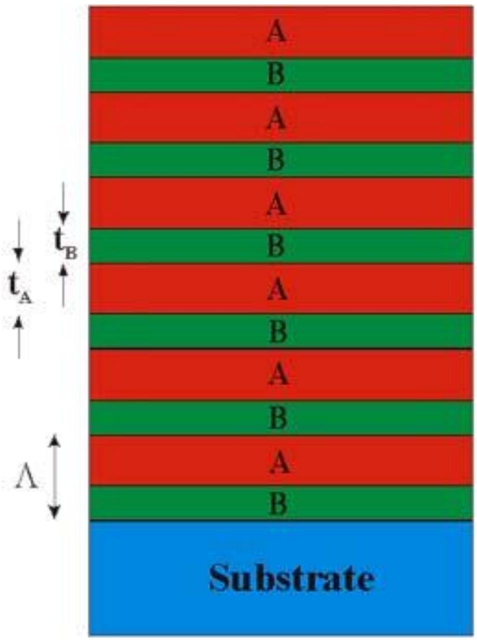
$$M = \begin{bmatrix} 0 & \frac{-j}{n_l} \\ -jn_l & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{-j}{n_h} \\ -jn_h & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{-j}{n_l} \\ -jn_l & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{-j}{n_h} \\ -jn_h & 0 \end{bmatrix} \cdots = \begin{bmatrix} \left(\frac{-n_h}{n_l}\right)^N & 0 \\ 0 & \left(\frac{-n_l}{n_h}\right)^N \end{bmatrix}$$

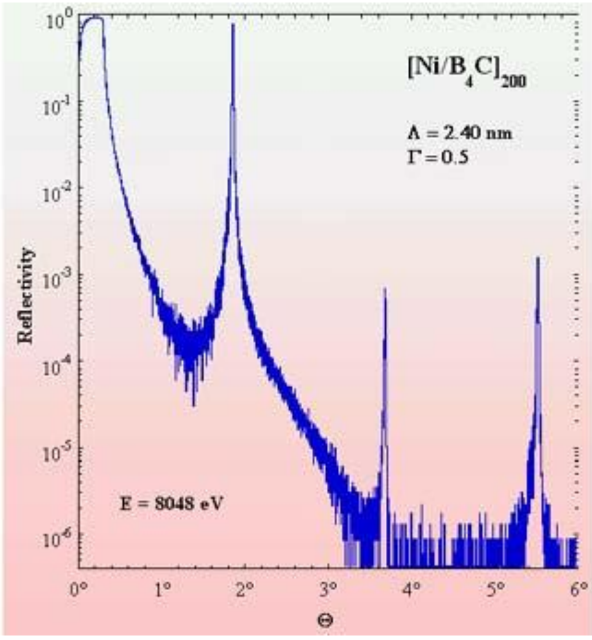
$$\Rightarrow \Gamma = \frac{(n_h/n_l)^{2N} - 1}{(n_h/n_l)^{2N} + 1} \rightarrow 1.$$

Eg. Determine the effective reflectances of an eight-layer stack (N=4) and thirty-layer stack (N=15) of ZnS (n_h=2.3) and MgF_2 (n_l=1.35).

(Sol.) (a) $|\Gamma|^2 = [\frac{(2.3/1.35)^8 - 1}{(2.3/1.35)^8 + 1}]^2 = 0.9409$, (b) $|\Gamma|^2 = [\frac{(2.3/1.35)^{30} - 1}{(2.3/1.35)^{30} + 1}]^2 = 0.999$

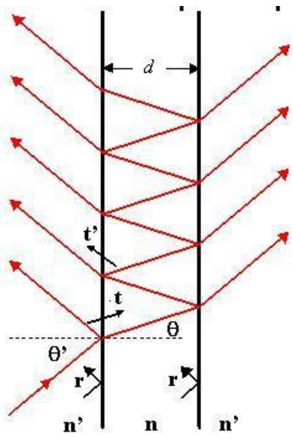
Eg. The relation between the reflectivity and the incidence angle.





3-9 Fabry-Perot Resonators

Fabry-Perot resonator: Lightwave is resonant between two parallel plates.



Path difference between two successive rays**:**

$$\frac{d}{\cos\theta}+\frac{d}{\cos\theta}\cdot\cos 2\theta = 2d\cos\theta$$

Total output E-field:

$$E_t = E_i tt' + E_i tt' rr' e^{j\delta} + E_i tt' r^2 r'^2 e^{j2\delta} + \cdots = \frac{E_i tt'}{1-rr'e^{j\delta}},$$

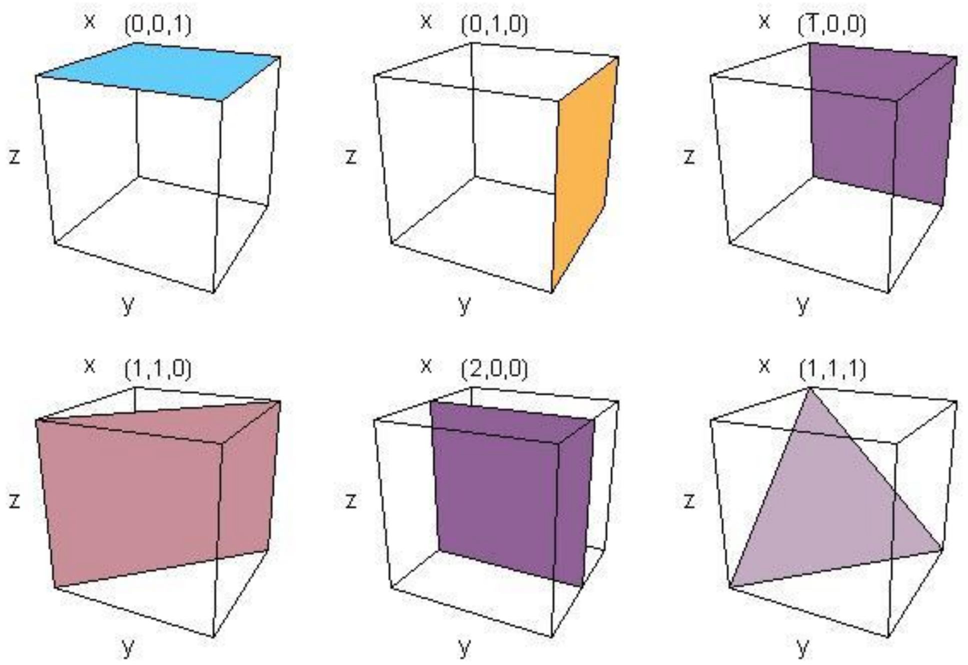
where $\delta = 2kd\cos\theta = \dfrac{4\pi nd\cos\theta}{\lambda_0}$ is the **optical phase difference**, and d is the thickness.

Total output intensity Fabry-Perot resonator:

$$I_t = \frac{I_i T^2}{\left|1-\mathrm{R}e^{j\delta}\right|^2} = \frac{I_i T^2}{(1-R)^2 + 4R\sin^2(\delta/2)},$$ where $R=rr'$, $T=tt'$, and $R+T+A$(absorption)=1.

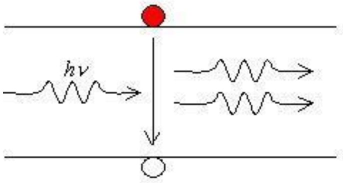
Natural cleavage planes: Semiconductor lasers usually utilize the natural cleavage planes as the both parallel mirrors of the Fabry-Perot resonator.

Miller indices: If a plane crossing (k,0,0), (0,h,0) and (0,0,l), then (k,h,l) is called Miller indices of the plane. **Note:** ($\bar{k}$,h,l) denotes (-k,h,l), (k,$\bar{h}$,l) denotes (k,-h,l), etc.



Note: GaAs's natural cleavage plane is (1,1,0)-plane. **Si**'s and **Ge**'s natural cleavage plane are (1,1,1)-plane.

3-10 Introduction to Lasers

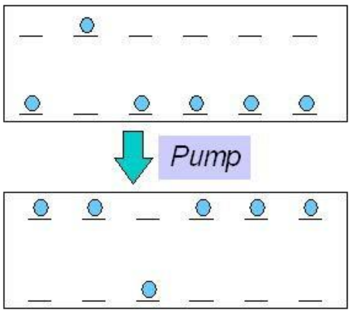


Stimulated emission: A photon induces an electron to fall down from a higher-energy level to a lower-energy level. And then it generates another photon with the same wavelength and the same phase.

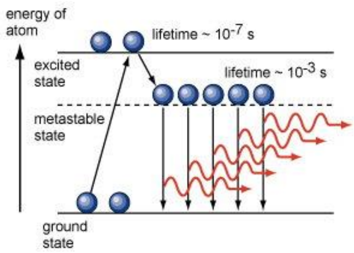
Basic characteristics of lasers:

1. Alignment.
2. Small broadening angle.
3. Single wavelength.
4. High Coherence.

Lasing conditions:

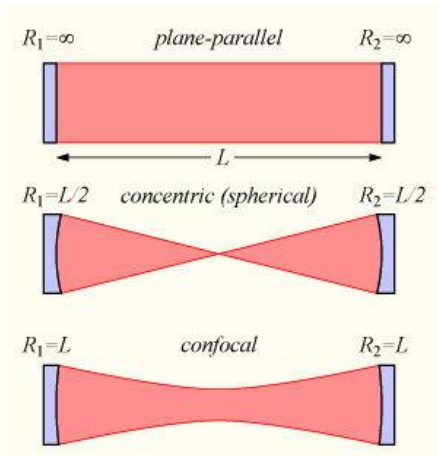


1. **Population inversion:** A certain higher-energy level has more electrons than a lower-energy level.



Eg. The energy-level system of a ruby laser: The population inversion occurs between the metastable state and the ground state.

2. **Pumping systems:** Utilizing current driving or other methods to pump electrons from a lower-energy level into a higher-energy level.
3. **Optical resonators:** The laser light traverses back and forth within an optical resonator to generate the stimulated emission.

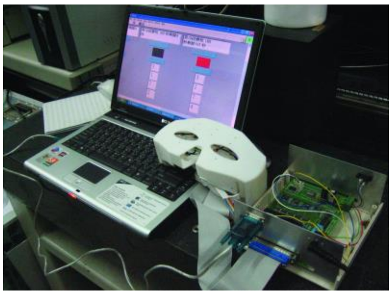


Application of lasers:

Laser Pointer

Laser Pickup Head

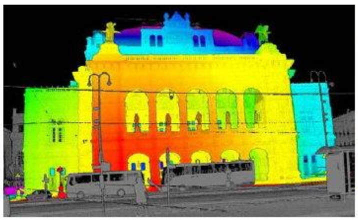
Laser Sight/Collimation



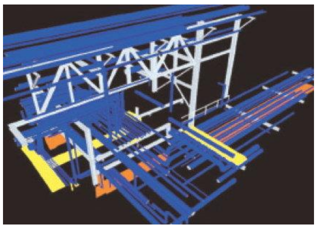
Laser knife in surgery

Laser acupuncture

Laser acupuncture machine

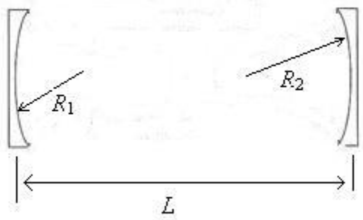


維也納歌劇院



Laser scanner and its applications

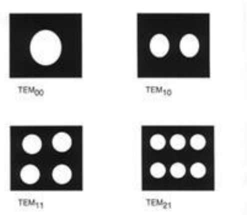
3-11 General Optical Resonators and Laser Modes



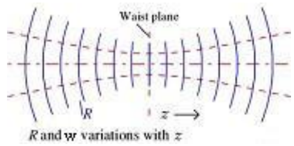
General laser resonator: Two mirrors with radii R_1 and R_2, separation L.

Stability condition: $0 \le (1-\frac{L}{R_1})(1-\frac{L}{R_2}) \le 1$

If a laser medium is situated within an unstable resonator, no stable laser can emit.



Laser modes: TEM_{mn} mode if m+1 spots in the horizontal direction and n+1 spots in the vertical direction.



Gaussian beam (TEM_{00} mode): Fundamental laser mode in an optical resonator or a lenslike medium.

$$\nabla^2\vec{E}+k^2\vec{E}=0 \Rightarrow (\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}+k^2)\vec{E}=(\nabla_t^{\,2}+\frac{\partial^2}{\partial z^2}+k^2)E=0\,.$$

Let $E=\psi(x,y,z)e^{-jkz}$ be a scalar field $\Rightarrow e^{-jkz}\nabla_t^{\,2}\psi+\frac{\partial}{\partial z}[\psi' e^{-jkz}-jk\psi e^{-jkz})+k^2\psi e^{-jkz}=0$,

where $\psi'=d\psi/dz$. Assume $k\psi' >> \psi'' << k^2\psi \Rightarrow \nabla_t^{2}\psi-2jk\psi'=0$.

Let $\psi=E_0\exp\{-j[P(z)+kr^2/2q(z)]\}$, where $r^2=x^2+y^2$.
Substitute $\psi=E_0\exp\{-j[P(z)+kr^2/2q(z)]\}$ into $\nabla_t^{2}\psi-2jk\psi'=0$
$\Rightarrow -(k/q)^2r^2-2j(k/q)-k^2r^2(1/q)'-2kP'=0$
$\Rightarrow (1/q)^2+(1/q)'=0$ and $P'=-j/q$.

Let $1/q=s'/s \Rightarrow s=az+b$, $q=q_0+z$, and $P'=-j/(z+q_0) \Rightarrow P=-j\ln(1+z/q_0)$.
Set $q_0=j\pi w_0^2n/\lambda$ and $k=2\pi n/\lambda \Rightarrow \psi=\exp\{-j[-j\ln(1+z/q_0)+kr^2/2(z+q_0)]\} \Rightarrow$

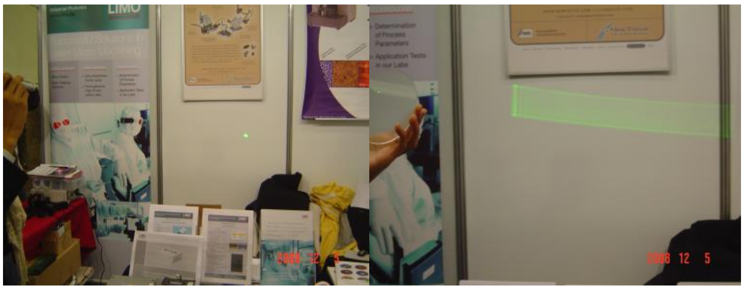
$E(x,y,z)=E_0\frac{w_0}{w(z)}\cdot\exp\left\{-j[kz-\eta(z)]-(x^2+y^2)\cdot[\frac{1}{w^2(z)}+\frac{jk}{2R(z)}]\right\}$, where $R(z)=z\left\{1+[\frac{\pi w_0^{\,2}n}{\lambda z}]^2\right\}$,

$w(z)=w_0\cdot\left\{1+[\frac{\lambda z}{\pi w_0^{\,2}n}]^2\right\}$, $\eta(z)=\tan^{-1}(\frac{\lambda z}{\pi w_0^{\,2}n})$, and $\frac{1}{q(z)}=\frac{1}{R(z)}-\frac{j\lambda}{\pi n w^2(z)}$.

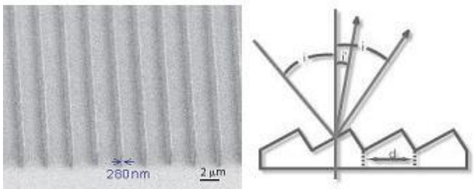
At **z=0**, waist: $w(0)=w_0$, $R(0)=\infty$, and $\eta(0)=0 \Rightarrow$ **$E(x,y,z=0)=E_0\exp[-(x^2+y^2)/w_0^2]$**

Beam divergent angle of Gaussian beam: $\theta_{beam}=\tan^{-1}[\frac{\lambda}{\pi w_0 n}]$.

Beam Shaper: Reshape laser beam and intensity distribution.

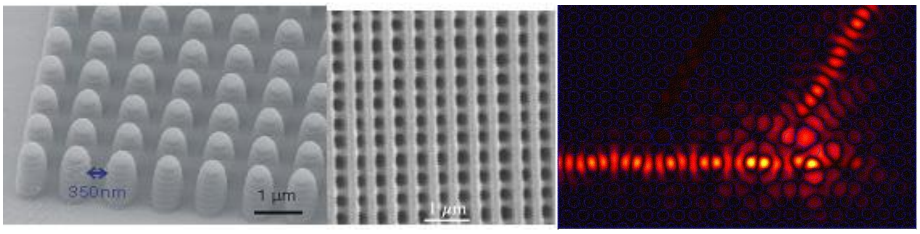


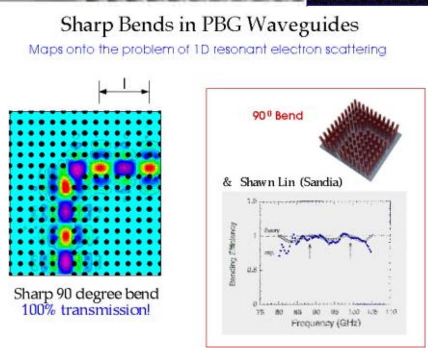
3-11 Gratings and Photonic Crystals



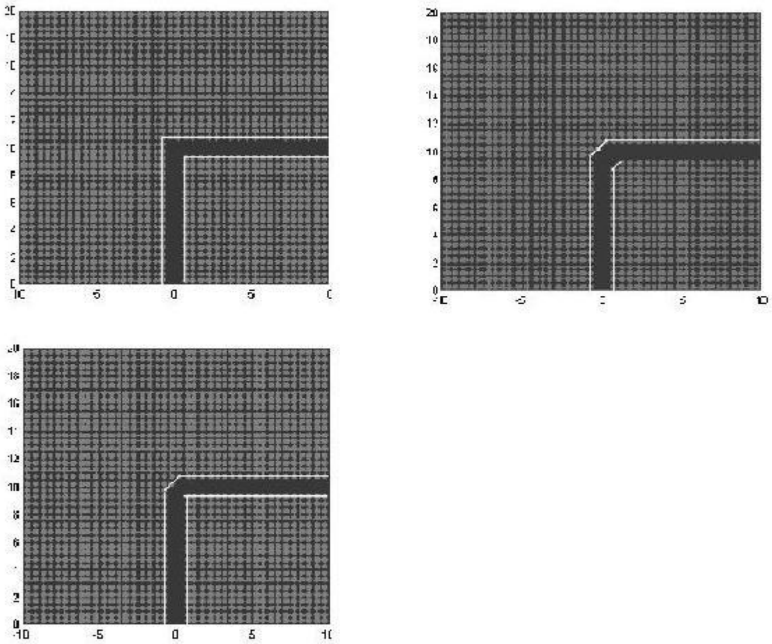
Grating: Periodical structure for optical diffraction.

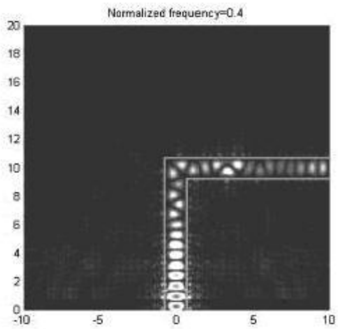
Photonic Crystals: Arrays of optical scatterers.



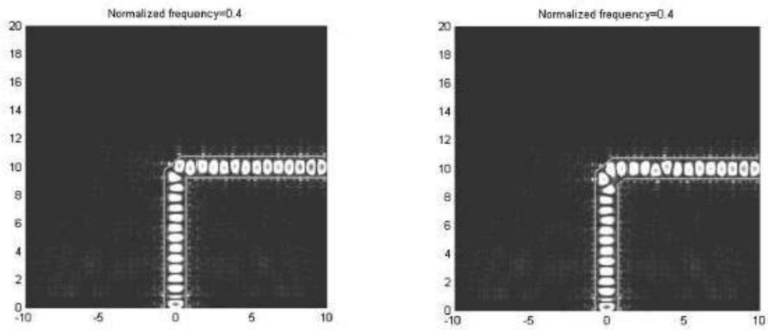


Eg. Comparison between 3 types of 90° bent photonic crystal waveguides. (by K. –Y. Lee, C. –C. Tsai 蔡佳辰, T. –C. Weng 翁宗誠, Y. –L. Kuo 郭奕麟, C. –W. Kao 高智偉, K. –Y. Chen 陳奎元, and Y. –J. Lin 林宇仁)





$a/\lambda = 0.4$, $w_0 = 0.5\mu m$



February 6, 2008

Dr. Keh-Yi Lee
Department of Electrical Engineering
Chinese Culture University
Taipei
Taiwan
Republic of China

Dear Dr. Lee,

Congratulations! Our research indicates that your article "Transmission Characteristics of 90-degree Bent Photonic Crystal Waveguides" published in *Fiber and Integrated Optics* volume 25, number 1 is the 3rd most-downloaded article published in 2006.

I would like to personally invite you to publish more of your work in *Fiber and Integrated Optics*. The journal is covered by all the appropriate indexing and abstracting services, including ISI/Thomson's *Science Citation Index* and *Current Contents*. As reported by Thomson, the 2006 impact factor for the journal is 0.600 (© 2007 Thomson Scientific, *Journal Citation Reports* ®). The journal has over 140 institutional and personal subscribers from six different regions and 33 different countries, and is also available online at an additional 560 institutions worldwide.

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If you have any questions regarding the relevance of a manuscript or any other questions about the journal, please feel free to contact me at henri.hodara@L-3com.com.

I wish you the best of luck in all your future research and publishing endeavors.

Sincerely,

Henri Hodara
Editor-in-Chief, *Fiber and Integrated Optics*