Group front evolution of Gaussian beam refracted from a right- to left-handed medium

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Abstract: Local enhancement of the power flow in a group front of a transient sinusoidal Gaussian beam refracted at the boundary of right- and left-handed media is observed. In vacuum the beam reaches a sinusoidal steady state that is a form of continuous wave (CW) Gaussian beam after 6 wave periods. Behind the interface plane in a low loss double negative medium with normal dispersion the individual Fourier components of the beam diffract at different angles and have diversified phase speeds. This results in the group front build-up that propagates on with the beam and moves sideways with respect to the group velocity direction, where energy is transported. The enhancement is illustrated with 2-D simulations using finite difference time domain (FDTD) method.

OCIS codes: (120.5710) Refraction; (160.4760) Optical properties; (260.0260) Physical optics; (260.2030) Dispersion; (260.2110) Electromagnetic theory; (350.5500) Propagation; (350.7420) Waves; (999.9999) Double negative materials; (999.9999) Metamaterials.

References and links

1. D. R. Smith and N. Kroll, "Negative refractive index in left-handed materials," Phys. Rev. Lett. **85**, 4184-4187 (2000).
2. R. W. Ziolkowski and E. Heyman, "Wave propagation in media having negative permittivity and permeability," Phys. Rev. E **64**, 056625-1-15 (2001).
3. R. A. Shelby, D. R. Smith and S. Schultz, "Experimental verification of a negative index of refraction," Science **292**, 77-79 (2001).
4. R. Ruppin, "Electromagnetic energy density in a dispersive and absorptive material," Phys. Lett. A **299**, 309-312 (2002).
5. J. Pacheco, Jr., T. M. Grzegorczyk, B.-I. Wu, Y. Zhang, and J. A. Kong, "Power propagation in homogeneous isotropic frequency-dispersive left-handed media," Phys. Rev. Lett. **89**, 257401-1-4 (2002).
6. D. R. Smith, D. Schurig, and J. B. Pendry, "Negative refraction of modulated electromagnetic waves," Appl. Phys. Lett. **81**, 2713-2715 (2002).
7. E. Cubukcu, K. Aydin, E. Ozbay, S. Foteinopoulou and C. M. Soukoulis, "Negative refraction by photonic crystals," Nature **423**, 604-605 (2003).
8. C. G. Parazzoli, R. B. Greegor, K. Li, B. E. C. Koltenbah, and M. Tanielian, "Experimental verification and simulation of negative index of refraction using Snell's law," Phys. Rev. Lett. **90**, 107401-1-4 (2003).
9. S. Foteinopoulou, E. N. Economou, and C. M. Soukoulis, "Refraction in media with a negative refractive index," Phys. Rev. Lett. **90**, 107402-1-4 (2003).
10. A. A. Houck, J. B. Brock, and I. L. Chuang, "Experimental observations of a left-handed material that obeys Snell's law," Phys. Rev. Lett. **90**, 137401-1-4 (2003).
11. S. A. Cummer. "Dynamics of causal beam refraction in negative refractive index materials," Appl. Phys. Lett. **82**, 2008-2010 (2003).
12. R. W. Ziolkowski and A. D. Kipple, "Causality and double-negative metamaterials," Phys. Rev. E **68**, 026615-1-9 (2003).
13. P. Kolinko and D. R. Smith, "Numerical study of electromagnetic waves interacting with negative index materials," Opt. Express **11**, 640-648 (2003).
14. R. W. Ziolkowski, "Pulsed and CW Gaussian beam interactions with double negative metamaterial slabs," Opt. Express **11**, 662-681 (2003).
15. D. R. Smith, P. Kolinko, and D. Schurig, "Negative refraction in indefinite media," J. Opt. Soc. Am. B **21**, 1032-1043 (2004).

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16. J. Garcia-Pomar and M. Nieto-Vesperinas, "Transmission study of prisms and slabs of lossy negative index media," Opt. Express **12**, 2081-2095 (2004).
17. S. Dutta Gupta, R. Arun and G. S. Agarval, "Subluminal and superluminal propagation in a left-handed medium," Phys. Rev. B **69**, 113104-1-4 (2004).
18. X. Huang and W. L. Schaich, "Wave packet propagation into a negative index medium," Am. J. Phys. **72**, 1232-1240 (2004).
19. W. T. Lu, J. B. Sokoloff, and S. Sridhar, "Refraction of electromagnetic energy for wave packets incident on a negative-index medium is always negative," Phys. Rev. E **69**, 026604-1-5 (2004).
20. Z. M. Zhang and K. Park, "On the group front and group velocity in a dispersive medium upon refraction from a nondispersive medium," J. Heat Transfer –Trans. ASME **126**, 244-249 (2004).
21. M. Scalora, G. D'Aguanno, N. Mattiucci, M.J. Bloemer, J.W. Haus, A.M. Zheltikov, "Negative refraction of ultra-short electromagnetic pulses," Appl. Phys. B **81**, 393-402 (2005).
22. H. Luo, W. Hu, X. Yi, H. Liu, J. Zhu, „Amphoteric refraction at the interface between isotropic and anisotropic media," Opt. Commun. **254**, 353-360 (2005).
23. I. S. Nefedov, A. J. Viitanen, and S. A. Tretyakov, "Electromagnetic wave refraction at an interface of a double wire medium," Phys. Rev. B **72**, 245113-1-9 (2005).
24. V. G. Veselago, "The electrodynamics of substances with simultaneously negative values of ε and µ," Sov. Phys. Usp. **10**, 509-514 (1968).
25. A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, 2nd ed. (Artech House, Norwood, MA 2000).
26. G. C. Sherman and K. E. Oughstun, "Energy-velocity description of pulse propagation in absorbing, dispersive dielectrics," J. Opt. Soc. Am. B **12**, 229-247 (1995).
27. G. D'Aguanno, M. Centini, M. Scalora, C. Sibilia, M. J. Bloemer, C. M. Bowden, J. W. Haus, and M. Bertolotti, "Group velocity, energy velocity, and superluminal propagation in finite photonic band-gap structures," Phys. Rev. E **63**, 036610 (2001).
28. C.-G. Huang and Y.-Z. Zhang, "Poynting vector, energy density, and energy velocity in an anomalous dispersion medium," Phys. Rev. A **65**, 015802 (2001).
29. M. Scalora, G. D'Aguanno, N. Mattiucci, N. Akozbek, M. J. Bloemer, M. Centini, C. Sibilia, and M. Bertolotti, "Pulse propagation, dispersion, and energy in magnetic materials," Phys. Rev. E **72**, 66601-1-8 (2005).
30. T. J. Cui and J. A. Kong, "Time-domain electromagnetic energy in a frequency-dispersive left-handed medium," Phys. Rev. B **70**, 205106-1-7 (2004).
31. S. A. Tretyakov, "Electromagnetic field energy density in artificial microwave materials with strong dispersion and loss," Phys. Lett. A **343**, 231–237 (2005).
32. A. D. Boardman and K. Marinov, "Electromagnetic energy in a dispersive metamaterial," Phys. Rev. B **73**, 165110-1-7 (2006).
33. Z. M. Thomas, T. M. Grzegorczyk, B. Wu, X. Chen, and J. A. Kong, "Design and measurement of a four-port device using metamaterials," Opt. Express **13**, 4737-4744 (2005).
34. E. B. Treacy, "Optical pulse compression with diffraction gratings," IEEE J. Quantum Electron. **5**, 454-458 (1969).
35. Zs. Bor and B. Racz, "Group velocity dispersion in prisms and its application to pulse compression and traveling-wave excitation," Opt. Commun. **54**, 165-169 (1985).
36. C. Radzewicz, M. J. La Grone and J. S. Krasinski, "Interferometric measurement of femtosecond pulse distortion by lenses," Opt. Commun. **126**, 185-190 (1996).
37. L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, (Pergamon, New York, 1960), pp. 253-256.

1. Introduction

Negative refraction of electromagnetic waves at the interface between a nondispersive right-handed material (RHM) and a strongly dispersive lossy left-handed medium (LHM) has received much attention in the literature of the last few years [1-23]. In most of those papers a left-handed material was a medium with effectively negative both electric permittivity and magnetic permeability as was proposed originally by Veselago [24]. The papers were addressed to demonstration of steady state phenomena such as negative refraction, energy propagation and focusing with double negative (DNG) slabs. However, the simulation tool which was the FDTD method is useful to observe transient effects as well [25].

Considerations on wave packet evolution during refraction were first published by Ziolkowski and Heyman [2]. Smith *et al.* [6] the first pointed out that interference fronts associated with plane wave modulated in space and frequency undergo positive refraction in LHM and move sideways with respect to the group velocity that is direction of negatively refracted energy flow. Then, Foteinopoulou *et al*. [9] wrote a remarkable sentence about refraction of a wave packet with both space and time Gaussian profiles on RHM-LHM interface: *The* wave is trapped temporarily at the interface, reorganizes, and, after a long time, the wave front moves eventually in the negative direction. That long reorganization time of the order of a few tens of the wave periods necessary to build up the new wavefront was attributed to resonant scattering behavior of the interface between a RHM and DNG photonic crystal. For a CW Gaussian beam the process of reorganization of wave front in a DNG material just behind the interface plane was observed by Cummer [11], who described it as a transient change of the refracted beam direction due to different angles of refraction of particular frequency components.

In 2004, Zhang and Park [20] made a clear distinction between group and group front velocity directions connected with propagation of spectral components of a Gaussian wave packet negatively refracted in a normal dispersion DNG medium. They confirmed the result of Smith *et al.* [6] that positively refracted group fronts move in a different direction than negatively refracted energy flow.

In the following text, a group front notion describes the initial part of an envelope of amplitudes of a group of interfering electromagnetic waves with different frequencies that in a dispersive medium travel with different speeds. After Zhang and Park [20], we accept that a group front moves with group front velocity, which is parallel to group velocity when a wave packet propagates in nondispersive medium, or when it incidents normally at the interface between RHM and dispersive LHM. When a wave packet incidents obliquely and refracts into LHM the group front and group velocities have different values and different directions. In Ref. [6] the group front velocity is called the velocity of interference front.

To avoid misinterpretation we indicate that when CW Gaussian beam propagating from RHM enters a normal dispersion LHM its group velocity has the same direction as the energy velocity. The difference between group velocity and energy velocity of light pulses in absorbing and dispersive dielectrics was discussed by Sherman and Oughstun [26], in finite photonic band-gap structures by D'Aguanno *et al*. [27], in anomalous dispersion media by Huang and Zhang [28] and in electrically and magnetically dispersive media by Scalora *et al*. [29]. In 2002 Rupin [4] described energy transport in LHM and gave expressions for energy velocity and energy density. The problem of electromagnetic energy stored in metamaterials continues to be a subject of interest [23, 27-32].

In the next two sections we present analysis of CW Gaussian beam refraction on a boundary of RHM and LHM with effective indices $n = 1$ and -1.5 for two cases of normal and oblique incidence. In FDTD animations we show modification and evolution of group fronts in a lossy and highly dispersive DNG medium.

Dependence of component values of refractive index tensor of DNG non-isotropic metamaterial on frequency may lead to engineering a distribution of energy in a refracted wave front. This might have important practical applications. Recently, a four-port device made of anisotropic LHM for selecting frequency bands was proposed [33]. In the past, group velocity dispersion was used for femtosecond pulse compression [34, 35]. Moreover, sideways motion of a group front of ultrashort light pulse with respect to the group velocity that is direction of energy flow was observed in interferometric measurement [36].

2. Group front velocity, group velocity and energy velocity

Zhang and Park [20] describe the group front velocity v_{gf} as the smallest speed at which the group front propagates in the direction normal to the group front and define it as follows

$$v_{gf} = \frac{\delta\omega}{\delta k}\frac{\delta\vec{k}}{\delta k}. \tag{1}$$

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According to Ziolkowski and Kipple [12] in low loss LHMs with normal dispersion the magnitude of group velocity v_g reaches

$$v_g(\omega) \approx \frac{\omega^2}{\omega^2 + \omega_p^2} c \tag{2}$$

and is in principle different from the energy propagation velocity v_e defined in LHM as [4]

$$v_e = \frac{\bar{S}}{\overline{W}} = \frac{-2c\,\mathrm{Re}\left(\sqrt{\varepsilon(\omega)/\mu(\omega)}\right)}{(\varepsilon' + 2\omega\varepsilon''/\Gamma_e) + (\mu' + 2\omega\mu''/\Gamma_m)\left|\varepsilon(\omega)/\mu(\omega)\right|}, \tag{3}$$

where minus means that in LHM energy velocity is antiparallel to the wave vector and $\bar{S}$ is obtained from the complex Poynting vector by

$$\bar{S} = \frac{1}{2}\mathrm{Re}\left\langle \vec{E} \times \vec{H}^* \right\rangle, \tag{4}$$

where brackets denote average over time. Ruppin [4] gives expressions for time averaged electromagnetic energy density $\overline{W}$ in a dispersive and absorptive LHM with complex permittivity ε and permeability μ

$$\overline{W} = \frac{\varepsilon_0}{4}\left(\varepsilon' + \frac{2\omega\varepsilon''}{\Gamma_e}\right)\left|\vec{E}\right|^2 + \frac{\mu_0}{4}\left(\mu' + \frac{2\omega\mu''}{\Gamma_m}\right)\left|\vec{H}\right|^2, \tag{5}$$

where superscripts denote real and imaginary parts of material parameters. In the case of vacuum the above formula reduces to

$$\overline{W} = \frac{1}{4}\left(\varepsilon_0\left|\vec{E}\right|^2 + \mu_0\left|\vec{H}\right|^2\right). \tag{6}$$

Recently, Scalora *et al*. [29] proved that in lossy media with normal dispersion the above equation is equivalent to the Landau expression [37] for energy density even for short pulses

$$\overline{W} = \frac{\varepsilon_0}{4}\mathrm{Re}\left(\frac{d[\omega\varepsilon(\omega)]}{d\omega}\right)\left|\vec{E}\right|^2 + \frac{\mu_0}{4}\mathrm{Re}\left(\frac{d[\omega\mu(\omega)]}{d\omega}\right)\left|\vec{H}\right|^2. \tag{7}$$

From eqs. (2) and (3), in our normal dispersion LHM with low loss described with Drude model $v_g = v_e = 8.57 \times 10^7$ m/s ≈ 0.29c. Dissipation of energy is too small to differentiate values of those velocities, because electric and magnetic dumping frequencies $\Gamma_e = \Gamma_m$ are 3 orders of magnitude smaller than corresponding plasma frequencies.

3. Normal incidence of CW Gaussian beam using FDTD method

So far observations of unusual behavior of CW Gaussian beams on a boundary between RHM and LHM addressed refraction and retardation effects and focusing with DNG slabs. We show that wave front of CW Gaussian beam that incidents normally on the interface undergoes local enhancement of the power flow and shape modification of envelope of amplitudes of a group of interfering waves with different frequencies.

In the FDTD simulation CW Gaussian beam of wavelength λ = 500 nm is generated in an empty 2D half-space and incidents at the interface between vacuum and a DNG material at 90° angle. The space of 10 × 20 μm is discretized in a square grid every 25 nm in x and z directions and simulations run for 5,000 time steps equal 5.89664e-17 s each. In vacuum CW Gaussian beam reaches a sinusoidal steady state after 6 wave periods, that is 120 time steps, thus it contains several wave components with different spatial and time frequencies. To control numerical stability of 2D simulation we accept the Courant number $S = 1/\sqrt{2}$ [25]. In the same way as in [12], the metamaterial electric permittivity $\varepsilon(\omega)$ and magnetic permeability $\mu(\omega)$ are described by lossy Drude model

$$\varepsilon(\omega) = 1 - \omega_{pe}^2 [\omega(\omega + i\Gamma_e)]^{-1} \text{ and } \mu(\omega) = 1 - \omega_{pm}^2 [\omega(\omega + i\Gamma_m)]^{-1}, \quad (8)$$

with equal electric and magnetic plasma frequencies $\omega_{pe} = \omega_{pm} = \omega_p = 2\pi f_0[1-(-n)]^{1/2} = 5.96 \times 10^{15}$ rad/s for f_0 = 600 THz and $n = -1.5$. Electric and magnetic dumping frequencies $\Gamma_e = \Gamma_m$ are equal and set 3 orders of magnitude smaller than corresponding plasma frequencies in LHM.

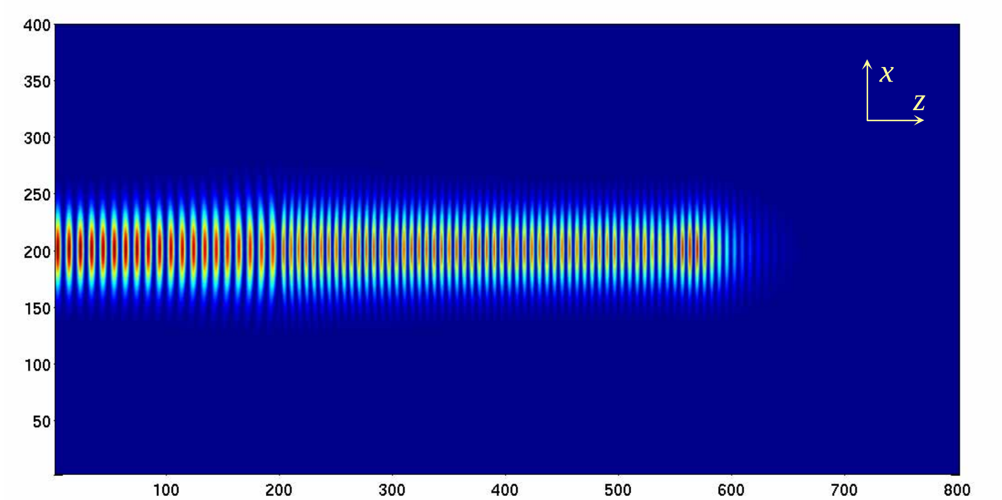

Fig. 1. (6.30 MB) CW Gaussian beam incidents normally at the interface (z = 200) between vacuum and LHM with $n = -1.5$. Evolution of the absolute value of Poynting vector $|\mathbf{E}\times\mathbf{H}|$, where **H** is parallel to the image plane, is shown in pseudocolors normalized to the maximum value in the incident beam.

Figure 1 shows propagation of CW Gaussian beam through an interface between vacuum with n = 1 and lossy $n = -1.5$ medium. Envelope of amplitudes of a group of interfering components with different frequencies reorganizes for about 8 wave periods at the interface. It is a simple reorganization of the wave vectors, which undergo a π phase shift as the nature of the medium changes. Then, after 50 periods, amplitude of the group front increases above that of incident wave and the enhanced group front propagates. There is no any time delay of light at the interface between two effective media with positive and negative n. Light is not trapped in the sense of being stopped, however, due to increase of $|n|$ value it is slowed down. In principle, light may be temporarily trapped just behind the interface between RHM and a photonic crystal with negative refractive index due to scattering, as it was observed in Ref. [9]. Due to negative refraction the diverging Gaussian beam changes into the converging one. Group front modification is better visible in Fig. 2 where evolution of the profile of Poynting vector is shown for low loss $n = -1.5$ medium. Normal incidence does not permit to distinguish the group velocity from the group front velocity.

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Fig. 2. (15.2 MB) Cross-section of the absolute value of Poynting vector for the propagation shown in Fig. 1. CW Gaussian beam incidents normally at the RHM-LHM interface indicated with the green line.

From the above simulation we find that the envelope of a transient sinusoidal Gaussian beam with steady state angular frequency $\omega = 2\pi f_0 = 3.77 \times 10^{15}$ rad/s propagates in the LHM with the group velocity $v_g = \partial\omega/\partial k$ approximately equal to (0.22±0.02)c. We observe that velocity of local enhancement of the power flow is different than the group velocity and equals (0.18±0.02)c. In case of normal incidence it is difficult to distinguish between the velocity of local enhancement of the power flow and the group front velocity also called the velocity of interference front and the energy velocity.

4. Oblique incidence of CW Gaussian beam using FDTD method

Figure 3 shows CW Gaussian beam of wavelength $\lambda = 500$ nm that incidents at the interface at 45° angle. The space of 10 × 15 μm is discretized in a square grid every 25 nm in x and z directions and simulations run for 2,500 time steps equal 5.89664e-17 s each.

In the case of oblique incidence of CW Gaussian beam at RHM-LHM interface we observe the following three effects illustrated in Fig. 3. Firstly, the negative refraction into DNG medium with $n = -1.5$ reported before in several papers [1-24]. Antiparallel group and phase velocities shown with red and blue vectors, respectively, are perpendicular to phase fronts denser than in vacuum. The direction of energy transport is parallel to the group velocity and normal to phase fronts. Secondly, modulation interference fronts that is group fronts move sideways with respect to the propagation direction of phase fronts as it was first observed by Smith *et al.* [6]. Clearly visible group fronts propagate in the positively refracted direction shown with violet vector. We first show the third effect of transient amplitude enhancement of the envelope of positively refracted group fronts of CW Gaussian beam. This purely dispersion effect results from different angles of refraction and diversified phase speeds of individual Fourier components of the Gaussian beam in LHM with $n = -1.5$. The group front build-up develops during about 50 wave periods behind the interface and propagates along several wavelengths.

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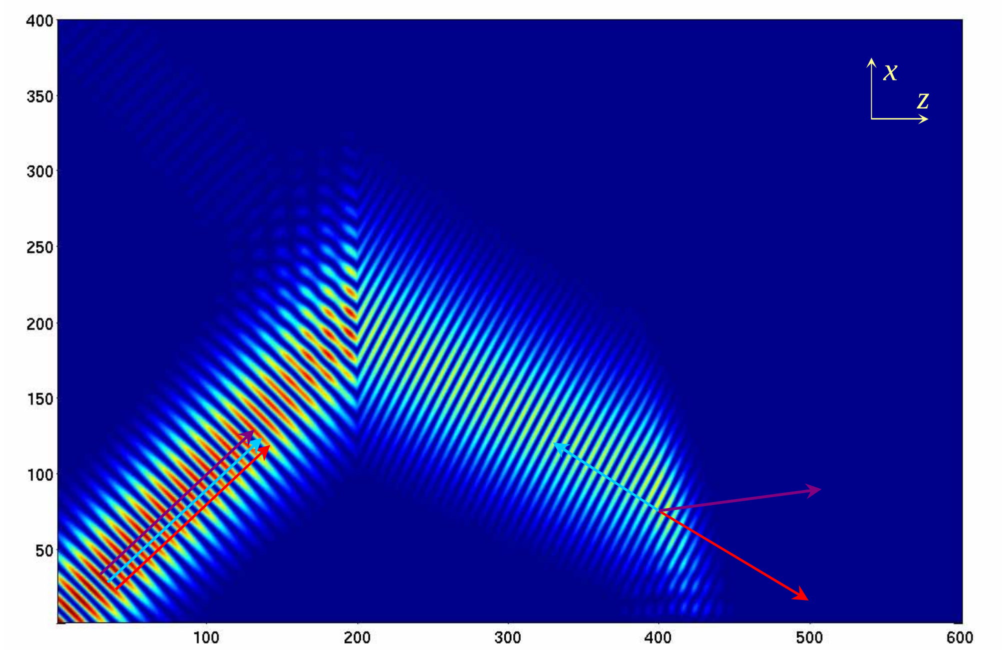

Fig. 3. (4.80 MB) CW Gaussian beam incidents at 45° at the interface ($z = 200$) between vacuum and DNG medium with $n = -1.5$. Pseudocolored evolution of the absolute value of Poynting vector $|\mathbf{E}\times\mathbf{H}|$, where **H** lays in the figure plane, is normalized to maximum value in the incident beam. In incident and refracted beams red, violet and blue vectors indicate group, group front, and phase velocities, respectively.

From the above simulation we find that the envelope of a transient sinusoidal Gaussian beam with steady state angular frequency $\omega = 2\pi f_0 = 3.77 \times 10^{15}$ rad/s propagates in the LHM with the group velocity $v_g = (0.19\pm0.02)$c that is close to that assessed in the normal incidence case. We observe that velocity of local enhancement of the power flow is parallel to the group front velocity and equals about (0.18±0.02)c. Oblique incidence results in clear separation of group and group front velocities.

Figure 4 shows CW Gaussian beam of wavelength $\lambda = 500$ nm that incidents at LHM slab at 45° angle. The space of 12.5 × 25 μm is discretized in a square grid every 25 nm in x and z directions and simulations run for 3,500 time steps equal 5.89664e-17 s each. We observe different temporal evolutions of the envelope of group fronts when the beam enters into LHM and RHM. In vacuum behind the slab enhancement of group front is preserved but decreases when the beam propagates on.

For LHM with effective $n = -1.5$ values of both ε' and μ' are negative and ε'' and μ'' are small and positive. As it was discussed by many authors density of electromagnetic energy in LHM with $|n| = 1.5$ is higher than in any other medium with $|n| < 1.5$.[4,29-32] Thus, average electromagnetic energy in the area limited in space and time in the group front build-up exceeds an average calculated in any other neighbor point within the beam. This results from high-pass filtering properties of the RHM-LHM interface which slows down certain frequencies.

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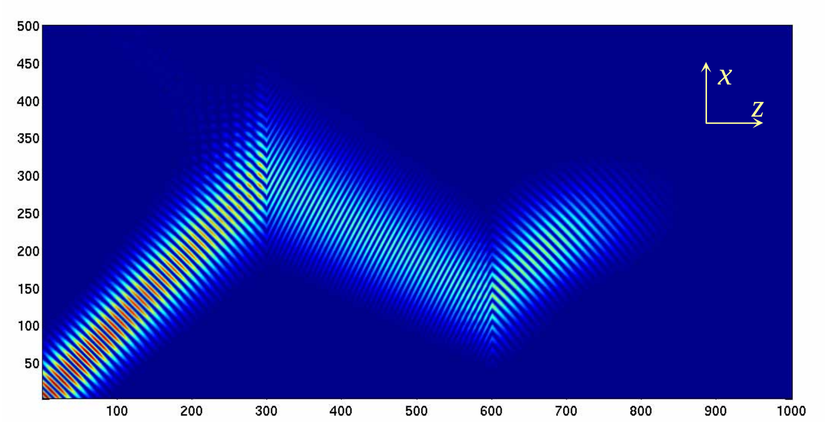

Fig. 4. (5.71 MB) CW Gaussian beam incidents at 45º from vacuum onto a plane-parallel plate made of DNG material with $n = -1.5$ and located between $z = 300$ and $z = 600$. Pseudocolored evolution of Poynting vector $|\mathbf{E}\times\mathbf{H}|$, where **H** lays in the figure plane, is normalized to maximum value in the incident beam.

5. Conclusion

Amplitude enhancement of the envelope of group fronts of CW Gaussian beam crossing the RHM-LHM interface is observed in FDTD simulations. For normal and oblique incidence the group front build-up develops during a few tens of wave periods behind the interface and propagates on in the direction of group velocity equal approx. 0.19÷0.22c. In the case of oblique incidence the values and directions of group front velocity v_{gf} = 0.18c and group velocity v_g = 0.19c are different. Positively refracted group front moves sideways with respect to negatively refracted phase fronts.

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